

# The Impact of Organizer Market Structure on Participant Entry Behavior in a Multi-Tournament Environment\*

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**Abstract.** A model of two tournaments, each with a field of two entrants is analyzed. Two high ability agents first decide which tournament to enter (with fields subsequently filled by low ability agents). The impact of organizer market structure on agent entry behavior and the resulting tournament fields is determined. If the marginal benefit of having high ability agents in an event is weakly increasing, a monopsonist organizer sets prizes so that the two high ability agents enter the same event. If this marginal benefit is diminishing, a monopsonist organizer sets prizes for which the high ability agents enter different events either: for all parameter values; or if and only if the difference in ability between high and low agents is small. Sequentially competing organizers set prizes for which both high ability agents enter the same event if and only if the marginal benefit of having two high ability agents in one event is relatively large. Further, with competing organizers there may be either a first or second mover advantage (depending upon which fields arise). Finally, Social Welfare may be higher or lower with competing organizers versus a monopsonist organizer, implying that greater organizer competition does not necessarily increase Social Welfare.

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# 1 Introduction

Labor market tournaments (in which payments to agents depend upon relative performance) have been examined extensively in the economics literature (pioneering works include Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Green and Stokey (1983), and Mookherjee (1984)). The primary focus has been an environment in which agents compete in a single tournament. However, in practice agents often have a choice over the competitive environment in which they will ultimately compete. The present study examines the entry decision by agents in a multi-tournament setting, with an ultimate focus on how organizer market structure impacts the choice of tournament prizes and resulting tournament fields.

The participation decision of agents and the resulting field of entrants in a single tournament has been examined previously. Lazear and Rosen (1981) identified an adverse selection problem arising when the organizer cannot observe the ability of agents and determines the field of a single tournament by randomly choosing a pair of agents from a common applicant pool. Agents will not sort themselves into applicant pools of different abilities, since: if there were a pool of high ability agents and a separate low ability pool, low ability agents would prefer to enter the high ability pool.

More recently, the endogenous entry decision of agents across multiple competitive environments of this type has been examined. Friebel and Matros (2005) examine a multi-tournament labor market contest in which heterogeneous firms (differing in bankruptcy probability) compete for workers by varying the magnitude of a pre-announced prize. Workers decide which firm to enter based upon the bankruptcy rates and prizes of each firm. The two primary insights are that in equilibrium: workers exert less effort in firms with higher bankruptcy rates; however, firms with higher bankruptcy rates may offer larger prizes. Mathews and Namoro (2008) consider a multi-tournament setting in which agents self select the competitive environment in which they will compete. The unique equilibrium outcome may be such that the tournament offering larger prizes attracts a field of lower quality

entrants than the tournament offering smaller prizes. Amegashie and Wu (2006) obtain a qualitatively similar result when examining the participation decision by agents of differing abilities over two all-pay auctions. They show that equilibria exist in which the highest ability agent chooses to compete for the less valued item. The present study distinguishes itself from these by treating the choice of prizes as endogenous, and focusing on how organizer market structure impacts the realized choice of prizes and entry behavior of tournament participants.

Beginning to think about the choice of prizes in a multi-tournament setting, first note that in general an agent would exert more effort when competing against rivals of relatively similar ability and would exert less effort when competing against rivals of relatively different ability. From here it follows that in order to maximize effort, an organizer would want to group agents of similar abilities in the same tournaments. However, in terms of providing general insights on the choice of prizes, this analysis is incomplete for at least two reasons. First, it explicitly assumes that the benefits to the organizer depend only upon effort and not upon the identity of tournament entrants (the model developed here allows the organizer's benefits to depend upon level of effort, identity of tournament participants, or both). Second, it implicitly assumes that the organizer can dictate which events agents compete in, or equivalently that agents have no choice over which events to compete in. In some instances, tournament participants choose which events to enter from a series of events. If this is the case, the entry decision should certainly depend upon the prizes in the different events.

Consider a multi-tournament environment in which agents self select the competitive environment in which they will compete. The primary focus is on how the tournament prizes and resulting tournament fields depend upon tournament organizer market structure. Suppose there are two tournaments, each with a field limited to two entrants. Agents are of two different ability levels (high ability and low ability), and the identity of tournament participants is potentially valued in and of itself by an organizer. First, two high

ability agents individually choose which event to enter, after which two low ability agents fill any remaining vacancies in the fields. In this setting, the resulting composition of entrants across events can be broadly thought of as either a “pooling” composition (in which both high ability agents compete in the same event) or a “separating” composition (in which the high ability agents compete in different events). The focus of the analysis is on the choice of prizes by the organizer(s) and the subsequent agent entry behavior. When setting prizes, an organizer must account for how the prizes will impact both the resulting tournament fields as well as the within tournament effort which agents will exert. Two alternative organizer market structures are considered: monopsony (in which a single organizer sets the prizes in both events) and sequential competition (in which two independent, competing organizers sequentially set prizes).

With a monopsonist organizer (whose benefits potentially depend upon the identities of tournament participants) either a pooling or a separating composition could be best, depending upon the behavior of the “marginal benefit of having high ability agents in a particular tournament.” If the marginal value of having high ability agents in a tournament is either constant or increasing, then a monopsonist organizer will set prizes for which both high ability agents enter the same event. If instead there is a diminishing marginal benefit from having high ability entrants in a particular event, a monopsonist organizer will either: always set prizes for which the high ability agents enter different events; or set prizes for which the high ability agents enter different events if and only if the difference in ability between the high ability agents and low ability agents is sufficiently small.

With competing organizers, we again have that either a pooling or a separating composition could result: a pooling composition will typically result when the marginal benefit of having high ability agents in a single event is relatively large; while a separating composition will typically result when the marginal benefit of having high ability agents in a single event is relatively small. Further, when competition between organizers will give

rise to a separating composition, it is shown that there is a second mover advantage in that the organizer choosing his prize first earns a smaller profit than the organizer choosing his prize second.<sup>1</sup>

Comparing the outcome across the alternate organizer market structures, it is shown that (depending upon the values of the parameters of the model) the high ability agents are: in some instances pooled in the same event regardless of organizer market structure, and in other instances separated across the two events regardless of organizer market structure. Further, there are parameter values for which the high ability agents are pooled by a monopsonist organizer but separated by competing organizers, and also there are parameter values for which the high ability agents are separated by a monopsonist organizer but pooled by competing organizers. Finally, Total Social Welfare may be either higher or lower with competing organizers versus a monopsonist organizer. That is, greater competition within the tournament organizer market does not necessarily lead to greater Social Welfare.

A model is developed and described in Section 2. Tournament participant behavior (both the within tournament choice of effort and the entry decision) is examined in Section 3. The choice of prizes by a monopsonist organizer is analyzed in Section 4, while the interaction between competing organizers is analyzed in Section 5. Comparisons between the outcome under a monopsonist organizer to the outcome with competing organizers are made in Section 6. Section 7 concludes.

## 2 Overview of Model

Consider a series of two rank order tournaments (“Event 1” and “Event 2”), each with a field limited to two entrants. Suppose there are two “high ability” agents ( $H$ ), each wishing to participate in one and only one event. After the type  $H$  agents decide which events to enter, the field of any tournament which

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<sup>1</sup>Numerical results suggest that there is instead a first mover advantage when competition leads to a pooling composition.

has not been filled will be filled by “low ability” agents ( $L$ ). As a result of this entry process, one of two compositions of entrants across the tournaments will result: a “pooling composition,” with both high ability agents in the same event; or a “separating composition,” with the high ability agents in different events.

First focusing on a monopsonist tournament organizer, conditions are determined specifying which type of composition will be implemented in order to maximize profit. Subsequently, an environment in which two independent tournament organizers compete by sequentially choosing prizes is analyzed. A comparison is made between these alternative environments.

The timing of the game is as follows:

1. prize levels for each event are set,<sup>2</sup>
2. tournament participants decide which events to enter,
3. competition takes place in each tournament (by way of participants exerting effort) and prizes are awarded.

This situation is analyzed recursively by first focusing on the tournament level competition, then analyzing the entry decision of tournament participants, and finally examining the choice of prizes.<sup>3</sup>

Let Event  $j$  denote a tournament with two entrants in which the first place finisher receives a prize of  $p_j$ , while the second place finisher receives nothing.

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<sup>2</sup>With a monopsonist organizer, a pair of prizes will be announced by the organizer at this stage. With competing organizers, first “Organizer  $l$ ” (i.e., the “leader”) announces a prize, after which “Organizer  $f$ ” (i.e., the “follower”) announces a prize.

<sup>3</sup>Many features of the model presented here are similar to features of the model analyzed by Mathews and Namoro (2008). Their focus is on the entry decision of tournament participants of three different skill levels over two tournaments, treating the prizes in tournaments as exogenous. The focus of the present study is on the strategic, endogenous choice of tournament prizes. In order to conduct this analysis, we presently assume a less general contest success function, tournament participants of only two ability levels (not three ability levels), and a second place prize of zero (assumptions which allow the choice of the organizer to be examined with greater ease).

Suppose the costs to a tournament organizer are simply equal to prizes paid, while the benefits to a tournament organizer depend upon (potentially) both the field of entrants as well as the level of effort exerted by tournament participants as follows. Let  $V_j \geq 0$  denote the value to an organizer of conducting a tournament with the field of entrants realized by Event  $j$  (this value depends upon only the identity of the entrants in Event  $j$  and not upon their levels of effort). Additionally, let  $E_j$  denote the total effort exerted by participants in Event  $j$ . Assume that total effort in Event  $j$  is valued according to  $r(E_j) = r\sqrt{E_j}$ , with  $r \geq 0$ .

The objective function of the organizer of a single Event  $j$  is thus

$$\gamma_j(p_j) = V_j + r\sqrt{E_j} - p_j,$$

while the objective function of a monopsonist organizer of both events is:

$$\gamma_M(p_1, p_2) = V_1 + V_2 + r\sqrt{E_1} + r\sqrt{E_2} - (p_1 + p_2).$$

The organizer's payoff depends critically upon the prizes across the events. This is true not only because the costs of organizing an event depends directly upon prizes, but additionally because the entry decision of tournament participants (and thus the realized  $(V_1, V_2)$  for a monopsonist organizer or the realized  $V_j$  for a competing organizer) and choice of effort level within each tournament (and therefore the resulting  $(E_1, E_2)$  or the resulting  $E_j$ ) depends upon prizes.

In order for an organizer to optimally set prizes, it is first necessary to determine how the tournament level choice of effort by each agent depends upon the tournament prize and the identity of the within tournament rival. Once this is done, the entry decision of the participants can be examined. Finally, the initial choice of organizer prizes will be analyzed (under the aforementioned alternative organizer market structures), taking the subsequent behavior of tournament participants as given.

### 3 Decisions of Tournament Participants

As noted, prizes influence two decisions made by tournament entrants: the decision of in which event to compete, and the decision of how much effort to exert. An analysis of these decisions is presented in this section.

#### 3.1 Tournament Level Competition

Consider a tournament in which entrants  $A$  and  $B$  compete by simultaneously choosing effort levels,  $e_A \geq 0$  and  $e_B \geq 0$ . Let  $\delta(e_A, e_B) = \frac{\delta e_A}{\delta e_A + (1-\delta)e_B}$  be the contest success function for agent  $A$ , specifying the probability with which  $A$  is the winner of the tournament when at least one agent chooses positive effort.<sup>4</sup> If both agents choose zero effort, define  $\delta(0, 0) = \delta$ . Supposing  $A$  is of (weakly) higher ability than  $B$ , consider  $\frac{1}{2} \leq \delta < 1$ . Assume the cost of exerting effort is simply equal to level of effort:  $c(e) = e$ .

The simultaneous choice of effort by  $A$  and  $B$  in the subgame consisting of a single tournament is analyzed in Appendix A. A unique pure strategy equilibrium is shown to exist in which  $e_A^* = e_B^* = e^* = p\delta(1 - \delta)$ . These effort levels result in  $\delta(e_A^*, e_B^*) = \delta$  and payoffs for  $A$  and  $B$  of:  $\Pi_{A,\{A,B\}} = p\delta^2$  and  $\Pi_{B,\{A,B\}} = p(1 - \delta)^2$ .

With agents of two different skill levels, a particular tournament realizes one of three fields of entrants:  $\{H, H\}$ ,  $\{H, L\}$ , or  $\{L, L\}$ . It is straightforward to apply the results above to each of these situations. For example, in an event with a field of either  $\{H, H\}$  or  $\{L, L\}$ , the tournament participants are of equal ability, so that  $\delta = \frac{1}{2}$ . Therefore, a participant in such a tournament exerts effort of  $\frac{1}{4}p$  and realizes a payoff of  $\frac{1}{4}p$ . In an event with a field of  $\{H, L\}$ ,  $\delta > \frac{1}{2}$  since the participants are of different abilities.<sup>5</sup> While agents  $H$  and  $L$  will exert a common level of effort,  $p\delta(1 - \delta)$ , they realize different payoffs of  $\Pi_{H,\{H,L\}} = p\delta^2$  and  $\Pi_{L,\{H,L\}} = p(1 - \delta)^2$  respectively.

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<sup>4</sup>Tournaments with logit-form contest success functions have been considered by Baik (1994), Hirshleifer (1989), and Rosen (1986).

<sup>5</sup>Henceforth  $\delta > \frac{1}{2}$  will specifically refer to the probability with which an agent of type  $H$  will be the winner in a tournament with a field of  $(H, L)$ .

### 3.2 Entry Decision of Tournament Participants

Suppose the two high ability entrants sequentially decide which event to enter. Let  $H_i$  and  $H_{ii}$  denote the two high ability agents, and suppose  $H_i$  is the agent who makes the initial entry decision. That is,  $H_i$  first decides to enter Event 1 or Event 2. After  $H_i$  makes this observable choice,  $H_{ii}$  then decides to enter Event 1 or Event 2. Finally, the field of any tournament that does not have two entrants is filled by low ability agents. Focusing on the entry decisions of the two high ability agents, a subgame perfect equilibrium will be determined for all acceptable values of  $\delta$  and values of prizes.<sup>6</sup>

Let  $p_1$  denote the larger and  $p_2$  denote the smaller of the prizes across the two events (i.e.,  $p_1 \geq p_2$ ). First note that if  $H_i$  enters Event 2, then  $H_{ii}$  has: an expected payoff of  $p_1\delta^2$  from instead entering Event 1; versus an expected payoff of  $\frac{1}{4}p_2$  from also entering Event 2. Since  $\delta^2 > \frac{1}{4}$  for any  $\delta > \frac{1}{2}$  and  $p_1 \geq p_2$ , it follows that if  $H_i$  enters Event 2, then  $H_{ii}$  will enter Event 1.

Next note that if  $H_i$  enters Event 1, then  $H_{ii}$  has: an expected payoff of  $\frac{1}{4}p_1$  from also entering Event 1; versus an expected payoff of  $p_2\delta^2$  from instead entering Event 2. From here two cases arise. First suppose  $\frac{1}{4}p_1 \geq \delta^2p_2$ . In this case, following a choice by  $H_i$  to enter Event 1,  $H_{ii}$  will enter Event 1.<sup>7</sup> The initial decision by  $H_i$  is now between competing against  $H_{ii}$  in Event 1 (leading to an expected payoff of  $\frac{1}{4}p_1$ ) or competing against a low ability agent in Event 2 (leading to an expected payoff of  $\delta^2p_2$ ). If  $\frac{1}{4}p_1 \geq \delta^2p_2$ , the former clearly gives a higher expected payoff than the latter, so that a “pooling composition” is realized in which both high ability agents enter Event 1. As a result, Event 1 realizes a field of  $(H, H)$  while Event 2 realizes a field of  $(L, L)$ .

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<sup>6</sup>As previously noted, Mathews and Namoro (2008) provide a detailed analysis of the specific factors influencing such a tournament entry decisions in a framework more general than the one considered here. The restrictions imposed here allow the tournament prizes to be endogenously determined while keeping the model tractable.

<sup>7</sup>In order to identify a unique choice by each agent at each decision node, it is assumed that if an agent has the same expected payoff in each tournament he will enter Event 1. Thus, for the present decision, if  $\frac{1}{4}p_1 = \delta^2p_2$  we assume  $H_{ii}$  enters Event 1.

Instead suppose  $\frac{1}{4}p_1 < \delta^2 p_2$ . Now, following a choice by  $H_i$  to enter Event 1,  $H_{ii}$  will enter Event 2. The decision of  $H_i$  is now one of competing against a low ability rival in either Event 1 or Event 2. Since  $p_1 \geq p_2$ ,  $H_i$  realizes a greater expected payoff from entering Event 1. Thus, for  $\frac{1}{4}p_1 < \delta^2 p_2$  a “separating composition” results in which one high ability agent enters each event. Thus, both Event 1 and Event 2 realize fields of  $(H, L)$ .

It is worth noting that if instead the entry process had been modelled as a simultaneous choice by the two high ability agents, essentially the same conditions would arise. Specifically, if  $\frac{1}{4}p_1 \geq \delta^2 p_2$ , then each agent has a dominant strategy of entering Event 1. Thus, the pooling composition arises. If instead  $\frac{1}{4}p_1 < \delta^2 p_2$ , then the best reply for each high ability agent (to any choice by his rival) is to enter the event that his rival does not enter. As such, there are two pure strategy equilibria, each characterized by a separating composition of entrants.<sup>8</sup>

Define  $\Omega(\delta) = \frac{1}{4\delta^2}$ . A separating composition (with each event realizing a field of  $(H, L)$ ) results if  $\frac{p_2}{p_1} > \Omega(\delta)$ ; a pooling composition (in which the event with larger prize attracts a field of  $(H, H)$  while the event with the smaller prize attracts a field of  $(L, L)$ ) results if  $\frac{p_2}{p_1} \leq \Omega(\delta)$ . Note that:  $\Omega\left(\frac{1}{2}\right) = 1$ ;  $\Omega(1) = \frac{1}{4}$ ; and  $\Omega'(\delta) = -\frac{1}{2\delta^3} < 0$ . Thus,  $\Omega(\delta) \in \left[\frac{1}{4}, 1\right)$  for all  $\delta \in \left(\frac{1}{2}, 1\right]$ . Since  $0 < \Omega(\delta) < 1$ , it follows that for any arbitrary  $p_1 > 0$ , there exists a unique  $\bar{p}_2 \in (0, p_1)$  such that: the separating composition results for  $p_2 \in (\bar{p}_2, p_1]$ , while the pooling composition results for  $p_2 \in [0, \bar{p}_2]$ .

From here the analysis will shift to the endogenous choice of tournament prizes. When choosing prize levels, it is assumed that the tournament en-

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<sup>8</sup>In this case there is also a mixed strategy equilibrium, in which each high ability agent randomizes between the two events. If these strategies were used, the two separating compositions would arise with positive probability, as would the pooling composition in which both enter Event 1. Further, a pooling composition in which both enter Event 2 also arises with positive probability. To proceed with a unique prediction of the resulting composition of entrants across the events, one of two equivalent (in terms of the predicted outcome) approaches could be taken: assume the entry decision of the high ability agents is sequential, or assume it is simultaneous and focus on pure strategy equilibria.

entrants subsequently choose which event to enter and how much effort to exert as specified thus far.

## 4 Monopsonist Tournament Organizer

First suppose there is a single, monopsonist organizer of the two events. The analysis of the choice by such an organizer proceeds as follows. First, the optimal prizes for realizing the separating composition are determined. Second, the optimal prizes for realizing the pooling composition are determined. In each of these cases, the resulting payoff of the organizer is determined. Finally, these payoffs are compared to each other, to determine if the organizer prefers to realize a pooling or separating composition. Recall that the objective function of a monopsonist organizer is:

$$\gamma_M(p_1, p_2) = V_1 + V_2 + r\sqrt{E_1} + r\sqrt{E_2} - (p_1 + p_2).$$

The monopsonist maximizes this expression by choosing  $p_1 \geq p_2 \geq 0$ , carefully accounting for the subsequent behavior of tournament entrants.

### 4.1 Optimal Prizes to Realize Separating Composition

To realize a field of  $(H, L)$  in each event, the chosen prizes must satisfy  $\frac{p_2}{p_1} > \Omega(\delta)$ . Letting  $V_{\{H,L\}}$  denote the value of having a field of  $(H, L)$ , we have  $V_1 = V_2 = V_{\{H,L\}}$  in this case. Further, each agent in Event 1 will exert effort of  $p_1\delta(1-\delta)$ , so that  $E_1 = 2p_1\delta(1-\delta)$ . Likewise, each agent in Event 2 exerts effort of  $p_2\delta(1-\delta)$ , so that  $E_2 = 2p_2\delta(1-\delta)$ . It follows that choosing prizes such that  $\frac{p_2}{p_1} > \Omega(\delta)$  gives the monopsonist organizer a payoff of

$$\gamma_{MS}(p_1, p_2) = 2V_{\{H,L\}} + r\sqrt{2p_1\delta(1-\delta)} + r\sqrt{2p_2\delta(1-\delta)} - (p_1 + p_2).$$

Lemma 1 characterizes the optimal prizes and resulting payoff for the monopsonist organizer in this case.

**Lemma 1.** For values of  $(p_1, p_2)$  satisfying  $p_1 \geq p_2$  and  $\frac{p_2}{p_1} > \Omega(\delta)$ ,  $\gamma_{MS}(p_1, p_2)$  is maximized by  $p_1^{MS*} = p_2^{MS*} = \frac{r^2\delta(1-\delta)}{2}$ . This choice results in  $\gamma_{MS}^* = 2V_{\{H,L\}} + r^2\delta(1-\delta)$ .

**Proof of Lemma 1.** Begin by noting that:  $\frac{\partial\gamma_{MS}}{\partial p_1} = r\sqrt{\frac{\delta(1-\delta)}{2p_1}} - 1$  and  $\frac{\partial\gamma_{MS}}{\partial p_2} = r\sqrt{\frac{\delta(1-\delta)}{2p_2}} - 1$ . From here,  $\frac{\partial^2\gamma_{MS}}{\partial p_1^2} < 0$  and  $\frac{\partial^2\gamma_{MS}}{\partial p_2^2} < 0$ .  
 $\frac{\partial\gamma_{MS}}{\partial p_1} = 0$  for  $p_1 = \frac{1}{2}\delta(1-\delta)r^2$ , and  $\frac{\partial\gamma_{MS}}{\partial p_2} = 0$  for  $p_2 = \frac{1}{2}\delta(1-\delta)r^2$ . The constraint of  $\Omega(\delta) < \frac{p_2}{p_1}$  is clearly satisfied at this pair of  $(p_1, p_2)$ , since  $\frac{p_2}{p_1} = 1$  while  $\Omega(\delta) \in [\frac{1}{4}, 1)$  in general. Therefore, in order to realize the separating composition, the optimal prizes are:  $p_1^{MS*} = p_2^{MS*} = \frac{r^2\delta(1-\delta)}{2}$ , resulting in  $\gamma_{MS}^* = 2V_{\{H,L\}} + r^2\delta(1-\delta)$ . *Q.E.D.*

It is clear that  $\gamma_{MS}^* = 2V_{\{H,L\}} + r^2\delta(1-\delta)$  is: increasing in  $V_{\{H,L\}}$ ; increasing in  $r$ ; and decreasing in  $\delta$  (since  $\delta \in (\frac{1}{2}, 1)$ ).

## 4.2 Optimal Prizes to Realize Pooling Composition

To realize the pooling composition (for which Event 1 attracts a field of  $(H, H)$ , while Event 2 attracts a field of  $(L, L)$ ), the prizes must be such that  $\frac{p_2}{p_1} \leq \Omega(\delta)$ . Let  $V_{\{H,H\}}$  denote the value to the organizer from a field of  $(H, H)$ ; let  $V_{\{L,L\}}$  denote the value to the organizer from a field of  $(L, L)$ . The relevant pooling composition result in  $V_1 = V_{\{H,H\}}$  and  $V_2 = V_{\{L,L\}}$ . Since each participant in each event is competing against a rival of identical ability, each entrant in Event 1 exerts effort of  $\frac{1}{4}p_1$  while each entrant in Event 2 exerts effort of  $\frac{1}{4}p_2$ . Thus,  $E_1 = \frac{1}{2}p_1$  and  $E_2 = \frac{1}{2}p_2$ . The resulting payoff of the organizer from choosing  $\frac{p_2}{p_1} \leq \Omega(\delta)$  is

$$\gamma_{MP}(p_1, p_2) = V_{\{H,H\}} + V_{\{L,L\}} + r\sqrt{\frac{p_1}{2}} + r\sqrt{\frac{p_2}{2}} - (p_1 + p_2).$$

Lemma 2 characterizes the optimal prizes and subsequent payoff of the monopsonist organizer in this case.

**Lemma 2.** For values of  $(p_1, p_2)$  satisfying  $\frac{p_2}{p_1} \leq \Omega(\delta)$ ,  $\gamma_{MP}(p_1, p_2)$  is maximized by  $p_1^{MP*} = \frac{r^2 \delta^2}{2} \left( \frac{1+2\delta}{1+4\delta^2} \right)^2$  and  $p_2^{MP*} = \frac{r^2}{8} \left( \frac{1+2\delta}{1+4\delta^2} \right)^2$ . This choice results in  $\gamma_{MP}^* = V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)}$ .

**Proof of Lemma 2.** First note that:

$$\frac{\partial \gamma_{MP}}{\partial p_1} = \frac{r}{2} \sqrt{\frac{1}{2p_1}} - 1 \quad (1)$$

and

$$\frac{\partial \gamma_{MP}}{\partial p_2} = \frac{r}{2} \sqrt{\frac{1}{2p_2}} - 1. \quad (2)$$

It immediately follows that  $\frac{\partial^2 \gamma_{MP}}{\partial p_1^2} < 0$  and  $\frac{\partial^2 \gamma_{MP}}{\partial p_2^2} < 0$ .

To have  $\frac{\partial \gamma_{MP}}{\partial p_1} = 0$  and  $\frac{\partial \gamma_{MP}}{\partial p_2} = 0$  simultaneously would require  $p_1 = p_2$ , implying that the constraint of  $\frac{p_2}{p_1} \leq \Omega(\alpha, \delta)$  is binding. Thus, the optimal  $(p_1, p_2)$  must satisfy  $p_2 = \Omega(\delta) p_1$  and  $-\frac{\frac{\partial \gamma_{MP}}{\partial p_1}}{\frac{\partial \gamma_{MP}}{\partial p_2}} = \Omega(\delta)$ , or equivalently:

$$p_2 = \frac{1}{4\delta^2} p_1 \quad (3)$$

and

$$-\frac{\partial \gamma_{MP}}{\partial p_1} = \frac{\partial \gamma_{MP}}{\partial p_2} \frac{1}{4\delta^2}. \quad (4)$$

Substituting (1), (2), and (3) into (4) yields:

$$-\left[ \frac{r}{2} \sqrt{\frac{1}{2p_1}} - 1 \right] = \left[ \frac{r}{2} \sqrt{\frac{2\delta^2}{p_1}} - 1 \right] \frac{1}{4\delta^2}.$$

Solving for  $p_1$  gives rise to

$$p_1^{MP*} = \frac{r^2 \delta^2}{2} \left( \frac{1+2\delta}{1+4\delta^2} \right)^2. \quad (5)$$

From (3) and (5) it follows that

$$p_2^{MP*} = \frac{r^2}{8} \left( \frac{1+2\delta}{1+4\delta^2} \right)^2. \quad (6)$$

The prizes from (5) and (6) give a payoff of  $\gamma_{MP}^* = V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)}$ .  
*Q.E.D.*

It is clear that  $\gamma_{MP}^* = V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)}$  is: increasing in  $V_{\{H,H\}}$ ; increasing in  $V_{\{L,L\}}$ ; increasing in  $r$ ; and decreasing in  $\delta$  (since  $\delta \in (\frac{1}{2}, 1)$ ).

### 4.3 Separating Composition or Pooling Composition?

The monopsonist organizer will: choose  $p_1^{MS*}$  and  $p_2^{MS*}$  (and realize the separating composition) if  $\gamma_{MS}^* > \gamma_{MP}^*$ ; and choose  $p_1^{MP*}$  and  $p_2^{MP*}$  (and realize the pooling composition) if  $\gamma_{MP}^* \geq \gamma_{MS}^*$ .<sup>9</sup>

Define

$$g(\delta) = \frac{(1+2\delta)^2}{8(1+4\delta^2)} - \delta(1-\delta)$$

and

$$\beta(V_{\{H,H\}}, V_{\{H,L\}}, V_{\{L,L\}}, r) = \frac{(V_{\{H,L\}} - V_{\{L,L\}}) - (V_{\{H,H\}} - V_{\{H,L\}})}{r^2}.$$

Theorem 1 describes the optimal choice by the monopsonist organizer.<sup>10</sup>

**Theorem 1.** *A monopsonist tournament organizer will choose prizes of:*

- i.  $p_1^{MS*}$  and  $p_2^{MS*}$  (and realize the separating composition) if  $g(\delta) < \beta$ ,
- ii.  $p_1^{MP*}$  and  $p_2^{MP*}$  (and realize the pooling composition) if  $g(\delta) \geq \beta$ .

**Proof of Theorem 1.** The monopsonist organizer will choose prizes leading to the pooling composition if and only if  $\gamma_{MP}^* \geq \gamma_{MS}^*$ , or equivalently

$$\begin{aligned} & V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)} \geq 2V_{\{H,L\}} + r^2\delta(1-\delta) \\ \Leftrightarrow & r^2 \left\{ \frac{(1+2\delta)^2}{8(1+4\delta^2)} - \delta(1-\delta) \right\} \geq 2V_{\{H,L\}} - (V_{\{H,H\}} + V_{\{L,L\}}) \\ \Leftrightarrow & g(\delta) \geq \frac{(V_{\{H,L\}} - V_{\{L,L\}}) - (V_{\{H,H\}} - V_{\{H,L\}})}{r^2} \\ \Leftrightarrow & g(\delta) \geq \beta. \end{aligned}$$

*Q.E.D.*

<sup>9</sup>It is assumed that the organizer will implement the pooling composition if  $\gamma_{MP}^* = \gamma_{MS}^*$ .

<sup>10</sup>In Theorem 1, and hereafter,  $\beta(V_{\{H,H\}}, V_{\{H,L\}}, V_{\{L,L\}}, r)$  is written as just  $\beta$ .

More insight into the result stated in Theorem 1 can be obtained by examining  $g(\delta)$  and  $\beta$ . Focusing first on  $g(\delta)$ , observe that  $g\left(\frac{1}{2}\right) = 0$  and  $g(1) = \frac{9}{40} = .225$ . Further,

$$g'(\delta) = (2\delta - 1) \left[ 1 - \frac{1 + 2\delta}{2(1 + 4\delta^2)^2} \right] = \frac{2\delta - 1}{2(1 + 4\delta^2)^2} \left[ (1 - \delta)^2 + 15\delta^2 + 32\delta^4 \right].$$

From here, clearly  $g'(\delta) > 0$  for all  $\delta \in \left(\frac{1}{2}, 1\right)$ , implying  $g(\delta) > 0$  for all  $\delta \in \left(\frac{1}{2}, 1\right)$ .

Focusing next on  $\beta$ , note that  $\beta$  is: increasing in  $V_{\{H,L\}}$ , but decreasing in  $V_{\{H,H\}}$  and  $V_{\{L,L\}}$ . Further,  $\beta$  is increasing in  $V_{\{H,L\}} - V_{\{L,L\}}$  and decreasing in  $V_{\{H,H\}} - V_{\{H,L\}}$ .<sup>11</sup> To recognize the impact of  $r$  on  $\beta$ , first observe that both  $\beta > 0$  and  $\beta < 0$  are possible. If  $\beta > 0$ , then  $\beta$  is decreasing in  $r$ . If instead  $\beta < 0$ , then  $\beta$  is increasing in  $r$  (i.e., for a larger value of  $r$ ,  $\beta$  is closer to zero in absolute terms but remains negative).<sup>12</sup>

Thus, with a monopsonist organizer market conditions are more conducive to realizing a pooling composition if (all other factors fixed):

1.  $\delta$  is larger (so the difference in abilities between  $H$  and  $L$  is greater, implying agents would exert less effort in an event with a field of  $(H, L)$ );
2.  $V_{\{H,H\}}$  is larger, which would imply that  $V_{\{H,H\}} - V_{\{H,L\}}$  is larger (so that the marginal benefit to the organizer of having a second high ability agent in a particular tournament is greater);
3.  $V_{\{L,L\}}$  is larger, which would imply that  $V_{\{H,L\}} - V_{\{L,L\}}$  is smaller (so that the marginal benefit to the organizer of having a first high ability agent in a particular tournament is smaller);
4.  $V_{\{H,L\}}$  is smaller, which would imply that  $V_{\{H,H\}} - V_{\{H,L\}}$  is larger (so that the marginal benefit to the organizer of having a second high ability

<sup>11</sup>Intuitively, both  $(V_{\{H,L\}} - V_{\{L,L\}})$  and  $(V_{\{H,H\}} - V_{\{H,L\}})$  should be non-negative.

<sup>12</sup>Recall,  $g(\delta) > 0$  for all  $\delta \in \left(\frac{1}{2}, 1\right)$ . Thus, if  $\beta < 0$ , then  $g(\delta) \geq \beta$  for all values of  $r$ . Therefore, a change in the value of  $r$  can possibly alter the relation between  $g(\delta)$  and  $\beta$  only when  $\beta > 0$  (in which case  $\beta$  is decreasing in  $r$ ). As a result, the only possible impact of a change in  $r$  on the relation between  $g(\delta)$  and  $\beta$  is the following: if initially  $g(\delta) < \beta$ , then a larger value of  $r$  may lead to  $g(\delta) \geq \beta$  instead of  $g(\delta) < \beta$ .

agent in a particular tournament is greater) and  $V_{\{H,L\}} - V_{\{L,L\}}$  is smaller (so that the marginal benefit to the organizer of having a first high ability agent in a particular tournament is smaller);

5.  $r$  is larger (so that the organizer values effort to a greater degree).

Focusing on the marginal value of having a high quality entrant in a particular field, two cases are possible. Either  $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$  (i.e., the marginal value of a high ability agent is diminishing), or  $V_{\{H,H\}} - V_{\{H,L\}} \geq V_{\{H,L\}} - V_{\{L,L\}}$  (i.e., the marginal value of a high ability agent is constant or increasing).

Standard economic intuition would suggest that this marginal benefit could reasonably be diminishing. This marginal benefit would be constant if the value of having a particular agent in an event does not at all depend upon which other entrants are in the event. This is the case in standard tournament models, which assume the benefits to the organizer depend only upon effort (so that  $V_{\{H,H\}} = V_{\{H,L\}} = V_{\{L,L\}} = 0$ ). Finally, this marginal benefit could be increasing if the value of different fields are such that there exists a synergy between the high ability agents, so that the value of having both high ability agents in the same event is greater than the sum of the values of having them in separate events.

The following corollaries relate the choice of the monopsonist organizer to the behavior of the marginal benefit of having high ability agents in an event.

**Corollary 1.** *If  $V_{\{H,H\}} - V_{\{H,L\}} \geq V_{\{H,L\}} - V_{\{L,L\}}$  (so that the marginal benefit of having high ability agents in an event is constant or increasing), then for all  $\delta \in (\frac{1}{2}, 1)$  a monopsonist organizer will choose prizes so that the pooling composition results.*

**Proof of Corollary 1.** If  $V_{\{H,H\}} - V_{\{H,L\}} \geq V_{\{H,L\}} - V_{\{L,L\}}$ , then  $\beta \leq 0$ . Since  $g(\delta) > 0$ , Theorem 1 implies that a monopsonist organizer will set prizes leading to the pooling composition. *Q.E.D.*

The result of Corollary 1 is intuitive. The organizer potentially values both effort and the identity of the participants in each event. Since agents exert greater effort when competing against rivals of relatively equal ability, effort is maximized by pairing the high ability agents with each other and pairing the low ability agents with each other. When the marginal benefit of having high ability agents in an event is constant or increasing, the portion of the organizer's payoff which depends upon the fields in the two events is also maximized by pairing the high ability agents in the same event.

**Corollary 2.** *If  $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$  (so that the marginal benefit of having high ability agents in an event is diminishing), then either:*

- i. for all  $\delta \in (\frac{1}{2}, 1)$  a monopsonist organizer will choose prizes so that the separating composition results, or*
- ii. there exists a unique  $\hat{\delta} \in (\frac{1}{2}, 1)$  such that a monopsonist organizer will choose prizes so that the separating composition results if and only if  $\delta < \hat{\delta}$ .*

**Proof of Corollary 2.** If  $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$ , then  $\beta > 0$ . Recall that  $g(\delta)$  is such that:  $g(\frac{1}{2}) = 0$ ,  $g(1) = \frac{9}{40} = .225$ , and  $g'(\delta) > 0$  for  $\delta \in (\frac{1}{2}, 1)$ . From here, two cases arise:  $\beta \geq \frac{9}{40}$  and  $\beta < \frac{9}{40}$ .

First consider  $\beta \geq \frac{9}{40}$ . In this case,  $g(\delta) < \beta$  for all  $\delta \in (\frac{1}{2}, 1)$ . Thus, Theorem 1 implies that the monopsonist organizer will choose prizes leading to the separating composition.

Next consider  $\beta < \frac{9}{40}$ . In this case,  $g(\frac{1}{2}) < \beta$  while  $g(1) > \beta$ . Since  $g'(\delta) > 0$ , it follows that there exists a unique  $\hat{\delta} \in (\frac{1}{2}, 1)$  such that  $g(\hat{\delta}) = \beta$ . From here:  $g(\delta) \geq \beta$ , for  $\delta \geq \hat{\delta}$ ; and  $g(\delta) < \beta$ , for  $\delta < \hat{\delta}$ . By Theorem 1, a monopsonist organizer will: choose prizes leading to the separating composition for  $\delta < \hat{\delta}$ ; and choose prizes leading to the pooling composition for  $\delta \geq \hat{\delta}$ . *Q.E.D.*

An implication of Corollary 2 is that if  $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$ ,

then the monopsonist organizer sets prizes so that the high ability agents enter different events for  $\delta$  sufficiently close to  $\frac{1}{2}$  (i.e., when  $H$  and  $L$  are of relatively equal ability). Intuitively this makes sense. When the marginal benefit of having high ability agents in an event is decreasing, then the part of the organizer's payoff which depends upon tournament fields is maximized by realizing a separating composition, with one high ability agent in each tournament. However, in comparison to a pooling composition, this composition makes agents exert less effort at the stage of tournament competition. The degree to which less effort is exerted under a separating composition (in comparison to a pooling composition) is smaller when  $\delta$  is smaller (and approaches zero as  $\delta \rightarrow \frac{1}{2}$ ). For  $\delta$  sufficiently close to  $\frac{1}{2}$ , the difference in effort becomes sufficiently small so that the decrease in payoff for the organizer from less effort (when realizing a separating composition) is less than the gain of  $2V_{HL} - (V_{HH} + V_{LL})$  (which is positive in this case) that arises from realizing a separating instead of a pooling composition.

Further, if  $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$ , then a monopsonist organizer will realize the separating composition for any  $\delta \in (\frac{1}{2}, 1)$  if the benefits to the organizer depend primarily upon the identities of the tournament participants as opposed to effort levels (that is, when  $r$  is sufficiently close to zero). To see this, note that when  $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$ , not only is  $\beta > 0$ , but  $\beta$  can be made arbitrarily large by sufficiently decreasing  $r$ . As a result, for sufficiently small  $r$  we have  $\beta \geq \frac{9}{40}$ , in which case  $g(\delta) < \beta$  for all  $\delta \in (\frac{1}{2}, 1)$  (so the organizer would choose to realize the separating composition for all possible  $\delta$ ). That is, if the value of high ability agents is diminishing, then a organizer who values effort to a sufficiently small degree will choose prizes so that the high ability agents enter different events.

## 5 Competing Tournament Organizers

Now suppose there are two tournament organizers competing with one another by sequentially choosing prizes. First, the organizer who is the "leader"

(denoted by  $l$ ) sets his prize, denoted  $p_l$ . After observing this choice, the organizer who is the “follower” (denoted by  $f$ ) sets his prize, denoted  $p_f$ . Once both prizes are known, the tournament entrants choose which events to enter and how much effort to exert as described and analyzed in Section 3.

Generally, the payoff of  $l$  is  $\gamma_l(p_l) = V_l + r\sqrt{E_l} - p_l$ , while the payoff of  $f$  is  $\gamma_f(p_f) = V_f + r\sqrt{E_f} - p_f$ . The choice of prizes by these competing organizers is analyzed recursively.

First consider the choice of  $p_f$  by  $f$ , after observing the value of  $p_l$  chosen by  $l$ . For every  $p_l > 0$ , there is a range of  $p_f$  which will give rise to each of the three different fields that  $f$  could realize. More precisely,  $f$  could opt to attract both low ability entrants by choosing  $p_f$  “sufficiently low” so that not only is  $p_f < p_l$  but further  $\frac{p_f}{p_l} \leq \Omega(\delta)$ . This would give  $f$  a payoff of

$$\gamma_f^{low}(p_f) = V_{\{L,L\}} + r\sqrt{\frac{p_f}{2}} - p_f.$$

If instead  $f$  chose a “mid-range prize” of  $p_f$  such that  $\Omega(\delta) < \frac{p_l}{p_f} < \frac{1}{\Omega(\delta)}$  (or equivalently  $p_l\Omega(\delta) < p_f < p_l\frac{1}{\Omega(\delta)}$ ), he would attract a field of  $(H, L)$  and realize a payoff of

$$\gamma_f^{mid}(p_f) = V_{\{H,L\}} + r\sqrt{2p_f\delta(1-\delta)} - p_f.$$

Finally,  $f$  could attract both high ability entrants by choosing  $p_f$  “sufficiently high” so that not only is  $p_f > p_l$  but further  $\frac{p_l}{p_f} \leq \Omega(\delta)$ . Such a prize of  $p_f \geq p_l\frac{1}{\Omega(\delta)}$  would give  $f$  a payoff of

$$\gamma_f^{high}(p_f) = V_{\{H,H\}} + r\sqrt{\frac{p_f}{2}} - p_f.$$

The payoff of  $l$  in each case can be defined in a similar manner. For instance, if  $l$  chose a value of  $p_l$  after which  $f$  would choose  $p_f$  such that  $p_f \geq p_l\frac{1}{\Omega(\delta)}$ , then  $l$  would realize a field of  $(L, L)$  and a payoff of  $\gamma_l(p_l, p_f) = V_{\{L,L\}} + r\sqrt{\frac{p_l}{2}} - p_l$ .

Returning attention to the payoff of  $f$ , note that  $\gamma_f^{low}(p_f)$ ,  $\gamma_f^{mid}(p_f)$ , and  $\gamma_f^{high}(p_f)$  are each concave functions of  $p_f$ . If there were no restrictions on the value of  $p_f$ , both  $\gamma_f^{low}(p_f)$  and  $\gamma_f^{high}(p_f)$  would be maximized by  $p_f = \frac{1}{8}r^2$ .

Likewise, without any restriction on  $p_f$ ,  $\gamma_f^{mid}(p_f)$  would be maximized by  $p_f = \frac{\delta(1-\delta)}{2}r^2$ . Since  $\delta \in \left(\frac{1}{2}, 1\right]$  it follows that  $\frac{\delta(1-\delta)}{2}r^2 < \frac{1}{8}r^2$ . That is, without accounting for how the value of  $p_f$  impacts the resulting tournament fields, we obtain the standard insight that an organizer would choose larger prizes in a tournament in which agents are of equal ability.

Even under the assumptions thus far, a general analysis of the sequential choice of prizes by competing organizers is not tractable. Therefore, the primary focus is on the results of a numerical analysis conducted as follows. First, accounting for the values of  $p_l\Omega(\delta)$  and  $p_l\frac{1}{\Omega(\delta)}$  which result for each possible  $p_l \geq 0$ , the payoff of the follower was determined for every possible  $p_f \geq 0$  according to the functions defined above. Second, for each possible  $p_l \geq 0$ , the optimal choice of  $p_f$  by  $f$  (that is, the prize that  $f$  would choose to maximize his own payoff) was determined. Viewing these values of  $p_f$  collectively, they represent  $p_f^{BR}(p_l)$ : the “best response function” for  $f$ , which specifies the optimal choice of  $p_f$  by  $f$  for every possible  $p_l$  that  $l$  could initially choose. From here, the payoff that  $l$  would ultimately realize for each possible  $p_l \geq 0$  (accounting for the subsequent choice of  $p_f$  by  $f$ ) was determined. This essentially gives us the payoff of  $l$  as a function of his own choice of prize, which can be denoted as:  $\gamma_l(p_l, p_f^{BR}(p_l))$ . Third,  $\gamma_l(p_l, p_f^{BR}(p_l))$  was maximized with respect to  $p_l$ , to determine the initial best choice of  $p_l$  by  $l$ .

After identifying the equilibrium  $p_l$  and  $p_f$ , it is straightforward to determine which entrants will enter which tournament, as well as the profit of both  $l$  and  $f$ . Further, the outcome with competing organizers can be compared to the outcome under a monopsonist organizer, in terms of the realized fields in the tournaments as well as Total Social Welfare (such comparisons are the focus of the discussion in Section 6).

Tables (1a)-(1c) present the results of the numerical analysis of the equilibrium when competing organizers sequentially choose prizes.<sup>13</sup> Each table

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<sup>13</sup>Numerical results were obtained for many more parameter values than those reported in Appendix B. All of the insights discussed in this subsection and the following section

individually focuses on fixed values of  $r$ ,  $\delta$ , and  $V_{LL}$ , while varying  $V_{HH}$  and  $V_{HL}$ . Within each cell in the main body of each table, the resulting tournament fields and ratio of organizer profits is reported. An entry of *LP* (for “leader, pooling”) indicates that in equilibrium the pooling composition in which  $l$  attracts both high quality entrants results. *CS* (for “competing, separating”) indicates that the equilibrium with competing organizers is such that the separating composition in which each event realizes a field of  $(H, L)$  arises. The reported numerical value in each cell is  $\frac{\gamma_l^*}{\gamma_f^*}$ , the ratio of the profit of  $l$  to the profit of  $f$  in equilibrium. For example, from Table (1a) we see that for  $r = 1$ ,  $\delta = .60$ ,  $V_{LL} = 1$ ,  $V_{HL} = 2.2$ , and  $V_{HH} = 4.8$ , these competing organizers will choose prizes for which:  $l$  attracts a field of  $(H, H)$ ,  $f$  attracts a field of  $(L, L)$ , and  $l$  earns a profit that is 2.1992 times greater than the profit of  $f$ .

Examining Tables (1a)-(1c) collectively we see that (all other factors fixed), a pooling composition tends to arise for smaller values of  $V_{HL}$  while a separating composition tends to arise for larger  $V_{HL}$ . To understand why this outcome results, note that for fixed  $V_{LL}$  and  $V_{HH}$ , a larger value of  $V_{HL}$  makes  $(V_{HH} - V_{HL})$  (i.e., the marginal value of a second high ability entrant) smaller and makes  $(V_{HL} - V_{LL})$  (i.e., the marginal value of a first high ability entrant) larger. Thus, for  $V_{HL}$  “sufficiently small” the relative benefit from having both high ability entrants in the same field is sufficiently large so that  $l$  finds it worthwhile to choose  $p_l$  large enough to attract both high ability entrants. The last row in each table indicates what “sufficiently small” means, by specifying the range of  $V_{HL}$  (for each  $V_{HH}$  considered) for which  $l$  will attract both high ability entrants.<sup>14</sup>

Focusing on the reported values of the equilibrium profit ratio, we have  $\frac{\gamma_l^*}{\gamma_f^*} < 1$  for parameter values for which *CS* results and  $\frac{\gamma_l^*}{\gamma_f^*} > 1$  for parameter values for which *LP* results. That is, there appears to be a “second mover

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hold true for these non-reported results as well.

<sup>14</sup>The larger value states the largest  $V_{HL}$  rounded to four decimal places for which *LP* arises. For example, from the last column of Table (1a) we have that for  $r = 1$ ,  $\delta = .60$ ,  $V_{LL} = 1$ , and  $V_{HH} = 7.2$ : *LP* arises for  $V_{HL} \leq 4.6193$ , while *CS* arises for  $V_{HL} \geq 4.6194$ .

advantage” ( $\gamma_f^* > \gamma_l^*$ ) for parameters such that  $CS$  will result, while there appears to be a “first mover advantage” ( $\gamma_l^* > \gamma_f^*$ ) for parameters such that  $LP$  will result. Proposition 1 characterizes the relation between  $\gamma_l^*$  and  $\gamma_f^*$  in instances in which competition between the organizers leads to the separating composition.

**Proposition 1.** *If the sequential choice of prizes by competing tournament organizers results in a separating composition, then there is a “second mover advantage” in that  $\gamma_f^* \geq \gamma_l^*$ .*

**Proof of Proposition 1.** Let  $p_l^*$  and  $p_f^*$  denote the chosen prizes and let  $\gamma_l^*$  and  $\gamma_f^*$  denote the resulting payoffs under the conjectured equilibrium. Focus on the choice of  $p_f$  by  $f$  following a choice of  $p_l = p_l^*$  by  $l$ . Recall that a separating composition will arise for any  $p_f \in \left(\Omega(\delta)p_l^*, \frac{1}{\Omega(\delta)}p_l^*\right)$ . Since  $\Omega(\delta) < 1$ , a choice of  $\bar{p}_f = p_l^*$  is clearly in this range. Further, a choice of  $\bar{p}_f = p_l^*$  would give  $f$  a payoff of  $\bar{\gamma}_f = \gamma_l^*$ . Thus, the optimal  $p_f^*$  must give  $f$  a profit that is at least as large:  $\gamma_f^* \geq \bar{\gamma}_f = \gamma_l^*$ . *Q.E.D.*

Continuing to focus on the profit ratio reported in Tables (1a)-(1c), observe that over the range of  $V_{HH}$  and  $V_{HL}$  leading to  $LP$ , this ratio appears to be: strictly increasing in  $V_{HH}$ ; and weakly decreasing in  $V_{HL}$ . To understand the first of these observations, recognize that we are considering an increase in  $V_{HH}$  over a range for which  $l$  attracts both high quality entrants (both before and after the increase in  $V_{HH}$ ). Such an increase in  $V_{HH}$  will not alter  $\gamma_f^*$ , but will strictly increase  $\gamma_l^*$ . Thus,  $\frac{\gamma_l^*}{\gamma_f^*}$  will strictly increase. Switching focus to the second of these observations, again recognize that we are considering an increase in  $V_{HL}$  when neither tournament attracts a field of  $(H, L)$ . Thus, this change can only impact the equilibrium profits indirectly, by possibly altering the prize which  $l$  must set to attract both high ability entrants. For  $V_{HL}$  sufficiently close to  $V_{LL}$ , the optimal prizes are not affected by an increase in  $V_{HL}$ , so that neither  $\gamma_l^*$  nor  $\gamma_f^*$  change. However, for  $V_{HL}$

sufficiently large, an increase in  $V_{HL}$  will make it that  $l$  must set a larger prize in order to make it not worthwhile for  $f$  to attract a field other than  $(L, L)$  to his own tournament. This makes  $\gamma_l^*$  smaller (while not changing  $\gamma_f^*$ ), leading to a decrease in  $\frac{\gamma_l^*}{\gamma_f^*}$ .

Over the range of  $V_{HH}$  and  $V_{HL}$  leading to  $CS$ , the profit ratio appears to be: strictly decreasing in  $V_{HH}$ ; and strictly increasing in  $V_{HL}$ . Note that when  $CS$  is relevant,  $l$  chooses the smallest possible  $p_l$  for which it is subsequently better for  $f$  to choose a prize for which  $f$  attracts a field of  $(H, L)$  as opposed to a field of  $(H, H)$ . Consider an increase in  $V_{HH}$  over a range for which both  $l$  and  $f$  attract fields of  $(H, L)$  (both before and after the increase in  $V_{HH}$ ). As  $V_{HH}$  increases,  $l$  must now choose a larger  $p_l$  in order to make it that  $f$  is still willing to ultimately accept a field of  $(H, L)$  at his event (as opposed to setting  $p_f$  to attract a field of  $(H, H)$ ). As a result of this choice of a larger  $p_l$  by  $l$ , the value of  $p_f$  ultimately chosen by  $f$  becomes larger as well. Together, these changes result in both  $\gamma_l^*$  and  $\gamma_f^*$  becoming smaller as  $V_{HH}$  increases. Further, based upon the numerical results, the decreases in profits appear to be such that  $\frac{\gamma_l^*}{\gamma_f^*}$  decreases. Now consider an increase in  $V_{HL}$  over a range for which both  $l$  and  $f$  attract fields of  $(H, L)$  (both before and after the increase in  $V_{HL}$ ). As  $V_{HL}$  increases,  $l$  can choose a smaller  $p_l$  in order to have it that  $f$  is willing to accept a field of  $(H, L)$  at his event (as opposed to setting  $p_f$  to attract a field of  $(H, H)$ ). As a result of this choice of a smaller  $p_l$  by  $l$ , the value of  $p_f$  ultimately chosen by  $f$  also decreases. Together, these changes result in increased values of both  $\gamma_l^*$  and  $\gamma_f^*$  as  $V_{HL}$  increases. Further, based upon the numerical results, the increases in profits appear to be such that  $\frac{\gamma_l^*}{\gamma_f^*}$  increases. Finally, observe that as the relevant outcome changes from  $CS$  to  $LP$  as a result of either an increase in  $V_{HH}$  or a decrease in  $V_{HL}$ , the profit ratio clearly increases (from less than 1, to greater than 1).

Comparing the results of Tables (1a)-(1c), insight can be gained into how the equilibrium tournament fields change as  $\delta$  changes. From the bottom row of these tables we see that for most of the reported values of  $V_{HH}$ , the largest value of  $V_{HL}$  for which  $LP$  arises decreases as  $\delta$  increases. However, this is

not always true, as this cutoff value of  $V_{HL}$  appears to be “u-shaped” for  $V_{HH} = 1.2$  (decreasing from 1.0937 to 1.0791 as  $\delta$  increases from .60 to .75, but then increasing from 1.0791 to 1.1309 as  $\delta$  increases from .75 to .90).<sup>15</sup>

## 6 Impact of Organizer Market Structure

To see how the equilibrium outcome depends upon organizer market structure, a comparison is made between the outcome with sequentially competing organizers to the outcome with a monopsonist organizer. The results reported in Tables (2a)-(2c) make this comparison. Again, each table individually focuses on fixed  $r$ ,  $\delta$ , and  $V_{LL}$ , while varying  $V_{HH}$  and  $V_{HL}$ . The following is reported within each cell in the main body of each table: the resulting fields with a monopsonist organizer, the resulting fields with competing organizers, and the ratio of Social Welfare with a monopsonist organizer to Social Welfare with competing organizers. An entry of *MP* (for “monopsonist, pooling”) indicates that a monopsonist organizer sets prizes for which both high ability agents are pooled into the event with larger prizes, while an entry of *MS* (for “monopsonist, separating”) indicates that a monopsonist organizer sets prizes for which the high ability agents are separated across the two events (*LP* and *CS* identify the resulting composition under competing organizers, as previously described).

In general, Social Welfare is defined as the sum of the payoff(s) of the organizer(s) of the tournaments and the four tournament entrants. If a separating composition is realized, Social Welfare is:

$$W_s = \left( V_{HL} + r\sqrt{2p_1\delta(1-\delta)} - p_1 \right) + \left( V_{HL} + r\sqrt{2p_2\delta(1-\delta)} - p_2 \right) + p_1\delta^2 + p_1(1-\delta)^2 + p_2\delta^2 + p_2(1-\delta)^2$$

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<sup>15</sup>Though not reported, results were obtained for different values of  $r$ , to see how the equilibrium depends upon this parameter. Generally, as  $r$  increases, the maximum  $V_{HL}$  leading to *LP* appears to be “u-shaped.” For example, with  $\delta = .75$ ,  $V_{LL} = 1$ , and  $V_{HH} = 4.8$ , the largest  $V_{HL}$  leading to *LP*: decreases from 2.5183 to 2.4139 as  $r$  increases from 0.2 to 1, but then increases from 2.4139 to 2.6141 as  $r$  increases from 1 to 5.

$$= 2V_{HL} + r\sqrt{2\delta(1-\delta)}(\sqrt{p_1} + \sqrt{p_2}) - 2\delta(1-\delta)(p_1 + p_2).$$

If a pooling composition is realized, Social Welfare is:

$$\begin{aligned} W_p &= \left( V_{HH} + r\sqrt{\frac{1}{2}p_1 - p_1} \right) + \left( V_{LL} + r\sqrt{\frac{1}{2}p_2 - p_2} \right) + \frac{1}{2}p_1 + \frac{1}{2}p_2 \\ &= V_{HH} + V_{LL} + r\sqrt{\frac{1}{2}}(\sqrt{p_1} + \sqrt{p_2}) - \frac{1}{2}(p_1 + p_2). \end{aligned}$$

Let  $W_m^*$  denote the equilibrium value of Social Welfare under a monopsonist organizer; let  $W_c^*$  denote the equilibrium value of Social Welfare under competing organizers. The value reported in each cell is  $\frac{W_m^*}{W_c^*}$ .

First note that comparing the realized fields across the alternative market structures, each of the four possibilities of  $(MP, LP)$ ,  $(MS, LP)$ ,  $(MP, CS)$ , and  $(MS, CS)$  can arise. In fact, each of these four outcomes can occur for common values of  $r$ ,  $\delta$ , and  $V_{LL}$ . For instance, from Table (2a) we see that for  $r = 1$ ,  $\delta = .60$ , and  $V_{LL} = 1$ :  $(MP, CS)$  arises for  $(V_{HL}, V_{HH}) = (1.1, 1.2)$ ;  $(MP, LP)$  arises for  $(V_{HL}, V_{HH}) = (1.1, 2.4)$ ;  $(MS, CS)$  arises for  $(V_{HL}, V_{HH}) = (2.2, 2.4)$ ; and  $(MS, LP)$  arises for  $(V_{HL}, V_{HH}) = (4.4, 7.2)$ . Generally,  $(MP, LP)$  arises (i.e., a pooling composition results for either organizer market structure) when  $V_{HL}$  is “relatively small,” while  $(MS, CS)$  arises (i.e., a separating composition results for either organizer market structure) when  $V_{HL}$  is “relatively large.” Further, for an intermediate range of  $V_{HL}$  the resulting fields differ for the two market structures considered. For instance, with  $r = 1$ ,  $\delta = .75$ , and  $V_{LL} = 1$  (the results for which are reported in Table (2b)) for each reported  $V_{HH}$  there is a range of  $V_{HL} \in (V_{LL}, V_{HH})$  such that a monopsonist sets prizes for which the high ability agents are pooled, while competing organizers set prizes for which the high ability agents are separated (this range of  $V_{HL}$  leading to  $(MP, CS)$  is reported in the last row of Table (2b)).

Shifting focus to the value of the Welfare Ratio, we can have either  $\frac{W_m^*}{W_c^*} > 1$  (i.e.,  $W_m^* > W_c^*$ ) or  $\frac{W_m^*}{W_c^*} < 1$  (i.e.,  $W_m^* < W_c^*$ ). That is, Social Welfare can be either larger or smaller with competing organizers as opposed

to a monopsonist organizer: increased competition within the tournament organizer market does not necessarily increase Social Welfare.

Further, the relation between  $W_m^*$  and  $W_c^*$  does not depend upon whether  $(MP, LP)$ ,  $(MS, LP)$ ,  $(MP, CS)$ , or  $(MS, CS)$  is relevant. For instance, from the first row of results in Table (2b), it is clear that we can have either  $\frac{W_m^*}{W_c^*} < 1$  or  $\frac{W_m^*}{W_c^*} > 1$  when  $(MP, LP)$  is relevant. Similarly, the results reported in the row corresponding to  $V_{HL} = 4.4$  in Table (2a) show that we can have either  $\frac{W_m^*}{W_c^*} < 1$  or  $\frac{W_m^*}{W_c^*} > 1$  for  $(MS, CS)$ . Likewise, from the results in Table (2b) we see that (for the corresponding values of  $r$ ,  $\delta$ , and  $V_{LL}$ )  $(MP, CS)$  arises for both  $(V_{HL}, V_{HH}) = (1.1, 1.2)$  and  $(V_{HL}, V_{HH}) = (2.2, 3.6)$ .  $\frac{W_m^*}{W_c^*} < 1$  for the former pair, while  $\frac{W_m^*}{W_c^*} > 1$  for the latter pair. Finally, as reported in Table (2a), (for the corresponding values of  $r$ ,  $\delta$ , and  $V_{LL}$ ) for  $(V_{HL}, V_{HH}) = (4.4, 7.2)$ ,  $(MS, LP)$  is relevant and  $\frac{W_m^*}{W_c^*} > 1$ . However, (although not reported in Appendix B)  $(MS, LP)$  is relevant but  $\frac{W_m^*}{W_c^*} < 1$  for  $r = 1$ ,  $\delta = .55$ ,  $V_{LL} = 1$ ,  $V_{HL} = 1.12$ , and  $V_{HH} = 1.2$ .

## 7 Conclusion

While labor market tournaments have been examined extensively in the economics literature, the primary focus has been on a single tournament environment. However, in practice agents often have a choice over the competitive environment in which they will ultimately compete. The present study examined a multi-tournament environment in which agents self select their competitive environment, with a focus on how organizer market structure impacts the outcome.

A monopsonist organizer may want to either pool or separate high ability agents, depending upon the behavior of the “marginal benefit of having high ability agents in a particular tournament.” If this marginal benefit is constant or increasing, then a monopsonist organizer sets prizes for which high ability agents enter the same event. If instead this marginal benefit is diminishing, then a monopsonist organizer either: always sets prizes for which the high

ability agents enter different events; or sets prizes for which the high ability agents enter different events if and only if the difference in ability between the high ability and low ability agents is sufficiently small.

With competing organizers, we again had that either a pooling or a separating composition could result: a pooling composition would typically result when the marginal benefit of having high ability agents in a single event is relatively large; while a separating composition would typically result when the marginal benefit of having high ability agents in a single event is relatively small. Further, when competition between organizers gives rise to a separating composition, there is a second mover advantage in that the organizer setting his prize first earns a smaller profit than the organizer setting his prize second.

Comparing the outcome across the alternate organizer market structures, it was shown that the high ability agents were: in some instances pooled in the same event regardless of organizer market structure, and in other instances separated across the two events regardless of organizer market structure. Further, for some parameter values the high ability agents were pooled by a monopsonist organizer but separated by competing organizers, while for other parameter values the high ability agents were separated by a monopsonist organizer but pooled by competing organizers. Finally, it was shown that Total Social Welfare could be either higher or lower with competing organizers versus a monopsonist organizer, implying that greater competition within the organizer market does not necessarily increase Social Welfare.

## Appendix A

When  $A$  and  $B$  compete for a prize of  $p$  as outlined in Section 3, their respective payoffs are:  $\Pi_A = p\delta(e_A, e_B) - e_A$  and  $\Pi_B = p[1 - \delta(e_A, e_B)] - e_B$ . With  $\delta(e_A, e_B) = \frac{\delta e_A}{\delta e_A + (1-\delta)e_B}$ :  $\frac{\partial \delta(e_A, e_B)}{\partial e_A} > 0$ ;  $\frac{\partial \delta(e_A, e_B)}{\partial e_B} < 0$ ;  $\frac{\partial^2 \delta(e_A, e_B)}{\partial e_A^2} < 0$ ; and  $\frac{\partial^2 \delta(e_A, e_B)}{\partial e_B^2} > 0$  (for  $e_A > 0$  and  $e_B > 0$ ). Recall, by definition,  $\delta(0, 0) = \delta$ .

There cannot be a pure strategy equilibrium with either agent choosing zero effort. To see this, consider the effort choice by  $i$  if his rival,  $-i$ , chose  $e_{-i} = 0$ . Choosing  $e_i = 0$  results in  $i$  winning with probability  $\delta < 1$ , while any  $e_i = \epsilon > 0$  results in  $i$  winning with probability one. Thus, for any  $\delta < 1$  and  $p > 0$ , the payoff for  $i$  from  $e_i = \epsilon > 0$  (when  $e_{-i} = 0$ ) is greater than that from  $e_i = 0$  for sufficiently small  $\epsilon$ . Therefore, a choice of zero effort by both agents is never an equilibrium. However, if  $e_{-i} = 0$  and  $e_i = \epsilon > 0$ ,  $i$  can increase his payoff by instead choosing  $e_i = \frac{1}{2}\epsilon$  (since  $i$  still always wins the prize of  $p$ , but now incurs lower effort costs). Therefore, there are no pure strategy equilibria with either agent exerting zero effort.

To find an equilibrium in which both agents exert positive effort, note:  $\frac{\partial \Pi_A}{\partial e_A} = p \frac{\partial \delta(e_A, e_B)}{\partial e_A} - 1$ ,  $\frac{\partial \Pi_B}{\partial e_B} = -p \frac{\partial \delta(e_A, e_B)}{\partial e_B} - 1$ ,  $\frac{\partial^2 \Pi_A}{\partial e_A^2} = p \frac{\partial^2 \delta(e_A, e_B)}{\partial e_A^2} < 0$ , and  $\frac{\partial^2 \Pi_B}{\partial e_B^2} = -p \frac{\partial^2 \delta(e_A, e_B)}{\partial e_B^2} < 0$ . At an equilibrium with  $e_A > 0$  and  $e_B > 0$ , both  $\frac{\partial \Pi_A}{\partial e_A} = 0$  and  $\frac{\partial \Pi_B}{\partial e_B} = 0$  must hold simultaneously. This requires  $\frac{\partial \delta(e_A, e_B)}{\partial e_A} = -\frac{\partial \delta(e_A, e_B)}{\partial e_B}$ , which for  $\delta(e_A, e_B) = \frac{\delta e_A}{\delta e_A + (1-\delta)e_B}$  requires  $e_A = e_B$  (i.e., in equilibrium the two agents exert equal effort). From here it readily follows (from either  $\frac{\partial \Pi_A}{\partial e_A} = 0$  or  $\frac{\partial \Pi_B}{\partial e_B} = 0$ ) that a unique pure strategy equilibrium exists in this subgame, with  $e_A^* = e_B^* = e^* = p\delta(1 - \delta)$ . These effort levels lead to payoffs for  $A$  and  $B$  of:  $\Pi_{A, \{A, B\}} = p\delta^2$  and  $\Pi_{B, \{A, B\}} = p(1 - \delta)^2$ .

## Appendix B - Numerical Results

**Table (1a).**  $r = 1, \delta = .60, V_{LL} = 1.$

$V_{HL}$	$V_{HH}$					
	1.2	2.4	3.6	4.8	6.0	7.2
1.1	(CS,0.9679)	(LP,1.4805)	(LP,1.8438)	(LP,2.1992)	(LP,2.5501)	(LP,2.8982)
2.2		(CS,0.9650)	(LP,1.6332)	(LP,2.1992)	(LP,2.5501)	(LP,2.8982)
3.3			(CS,0.9638)	(CS,0.7495)	(LP,2.3026)	(LP,2.8982)
4.4				(CS,0.9631)	(CS,0.8119)	(LP,1.9178)
5.5					(CS,0.9626)	(CS,0.8458)
6.6						(CS,0.9623)
(LP)	(1,1.0937]	(1,1.7688]	(1,2.4719]	(1,3.1836]	(1,3.8999]	(1,4.6193]

**Table (1b).**  $r = 1, \delta = .75, V_{LL} = 1.$

$V_{HL}$	$V_{HH}$					
	1.2	2.4	3.6	4.8	6.0	7.2
1.1	(CS,0.9756)	(LP,1.8443)	(LP,2.4968)	(LP,3.1365)	(LP,3.7691)	(LP,4.3972)
2.2		(CS,0.9739)	(CS,0.7988)	(LP,1.7255)	(LP,2.7922)	(LP,3.8589)
3.3			(CS,0.9733)	(CS,0.8572)	(CS,0.7234)	(LP,1.5390)
4.4				(CS,0.9730)	(CS,0.8861)	(CS,0.7894)
5.5					(CS,0.9728)	(CS,0.9033)
6.6						(CS,0.9726)
(LP)	(1,1.0791]	(1,1.5030]	(1,1.9543]	(1,2.4139]	(1,2.8780]	(1,3.3451]

**Table (1c).**  $r = 1, \delta = .90, V_{LL} = 1.$

$V_{HL}$	$V_{HH}$					
	1.2	2.4	3.6	4.8	6.0	7.2
1.1	(LP,1.1067)	(LP,2.0220)	(LP,2.8261)	(LP,3.6158)	(LP,4.3978)	(LP,5.1746)
2.2		(CS,0.9666)	(CS,0.8245)	(CS,0.6730)	(LP,2.1223)	(LP,3.1890)
3.3			(CS,0.9697)	(CS,0.8757)	(CS,0.7782)	(CS,0.6747)
4.4				(CS,0.9713)	(CS,0.9012)	(CS,0.8293)
5.5					(CS,0.9723)	(CS,0.9165)
6.6						(CS,0.9730)
(LP)	(1,1.1309]	(1,1.4678]	(1,1.8108]	(1,2.1555]	(1,2.5012]	(1,2.8475]

**Table (2a).**  $r = 1, \delta = .60, V_{LL} = 1.$ 

$V_{HL}$	$V_{HH}$					
	1.2	2.4	3.6	4.8	6.0	7.2
1.1	(MP,CS,0.9645)	(MP,LP,1.0269)	(MP,LP,1.0799)	(MP,LP,1.1215)	(MP,LP,1.1546)	(MP,LP,1.1814)
2.2		(MS,CS,0.9736)	(MP,LP,1.1007)	(MP,LP,1.1215)	(MP,LP,1.1546)	(MP,LP,1.1814)
3.3			(MS,CS,0.9819)	(MS,CS,1.0659)	(MP,LP,1.1753)	(MP,LP,1.1814)
4.4				(MS,CS,0.9880)	(MS,CS,1.0571)	(MS,LP,1.3488)
5.5					(MS,CS,0.9926)	(MS,CS,1.0519)
6.6						(MS,CS,0.9963)
(MP,CS)	[1.0938, 1.1039]	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }
(MS,LP)	{ $\emptyset$ }	[1.7040, 1.7688]	[2.3040, 2.4719]	[2.9040, 3.1836]	[3.5040, 3.8999]	[4.1040, 4.6193]

**Table (2b).**  $r = 1, \delta = .75, V_{LL} = 1.$ 

$V_{HL}$	$V_{HH}$					
	1.2	2.4	3.6	4.8	6.0	7.2
1.1	(MP,CS,0.9988)	(MP,LP,0.9930)	(MP,LP,1.0217)	(MP,LP,1.0451)	(MP,LP,1.0640)	(MP,LP,1.0794)
2.2		(MS,CS,0.9799)	(MP,CS,1.0185)	(MP,LP,1.1637)	(MP,LP,1.1335)	(MP,LP,1.1127)
3.3			(MS,CS,0.9826)	(MS,CS,0.9772)	(MP,CS,1.0577)	(MP,LP,1.2942)
4.4				(MS,CS,0.9848)	(MS,CS,0.9833)	(MS,CS,0.9954)
5.5					(MS,CS,0.9866)	(MS,CS,0.9870)
6.6						(MS,CS,0.9881)
(MP,CS)	[1.0792, 1.1264]	[1.5031, 1.7264]	[1.9544, 2.3264]	[2.4140, 2.9264]	[2.8781, 3.5264]	[3.3452, 4.1264]
(MS,LP)	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }

**Table (2c).**  $r = 1, \delta = .90, V_{LL} = 1.$ 

$V_{HL}$	$V_{HH}$					
	1.2	2.4	3.6	4.8	6.0	7.2
1.1	(MP,LP,0.9665)	(MP,LP,0.9791)	(MP,LP,0.9966)	(MP,LP,1.0120)	(MP,LP,1.0247)	(MP,LP,1.0351)
2.2		(MS,CS,0.9777)	(MP,CS,1.0304)	(MP,CS,1.2685)	(MP,LP,1.1869)	(MP,LP,1.1566)
3.3			(MS,CS,0.9819)	(MS,CS,0.9654)	(MP,CS,1.0424)	(MP,CS,1.2103)
4.4				(MS,CS,0.9844)	(MS,CS,0.9731)	(MS,CS,0.9691)
5.5					(MS,CS,0.9861)	(MS,CS,0.9779)
6.6						(MS,CS,0.9874)
(MP,CS)	[1.1310, 1.1705]	[1.4679, 1.7705]	[1.8109, 2.3705]	[2.1556, 2.9705]	[2.5013, 3.5705]	[2.8476, 4.1705]
(MS,LP)	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }	{ $\emptyset$ }

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