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PROBABILISTIC DOMINANCE AND STATUS QUO BIAS

GIL RIELLA[†] AND ROEE TEPER[‡]

Abstract. Decision makers have a strong tendency to retain the current state of affairs. This well-documented phenomenon is termed status-quo bias. We present the *probabilistic dominance* approach to status-quo bias: an alternative is considered acceptable to replace the status quo only if the chances of a (subjectively) severe loss, relative to the status quo, are not too high. Probabilistic dominance is applied and behaviorally characterized in a choice model that allows for a range of status quo biases, general enough to accommodate unanimity, but also standard expected utility maximization. We present a comparative notion of "revealing more bias towards the status quo" and study its implications to the probabilistic dominance model of choice. Lastly, the model is applied to the endowment effect phenomenon and to a problem of international portfolio choice when investors are home biased.

Keywords: Probabilistic dominance, status quo bias, comparative status quo bias, endowment effect, home bias.

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1. Introduction

Standard models of choice describe a decision maker who ought to choose one or a few elements from a collection of alternatives. Usually, payoff-irrelevant factors such as an initial endowment or a default reference point are considered irrelevant to the rational assessment of the alternatives. However, a growing amount of empirical data suggests that such factors affect behavior and that standard models are insufficient to describe real-life decision-making processes. Moreover, it is well-established by now that individuals usually have a strong tendency to retain the current state of affairs. This phenomenon is traditionally referred to as *status quo bias* (Samuelson and Zeckhauser (1988)).

A recurrent explanation to status quo bias and related phenomena is anticipated regret. In particular, experimental findings point to a distinction between the status quo and new alternatives—sensation of regret is stronger when bad outcomes are a result of choosing a new alternative than when they result from retaining the status quo (e.g., Kahneman and Tversky (1982), Inman and Zeelenberg (2002) and Zeelenberg, van den Bos, van Dijk, and Pieters (2002)).

A prominent approach involving status quo considerations is that of *unanimity*: an option is considered a candidate to replace the status quo if and only if it is better than the status quo with respect to every criterion in a collection of criteria. The collection of criteria is determined subjectively by the decision maker and according to the nature of the decision problem in study. For example, Bossert and Sprumont (2003) as well as Masatlioglu and Ok (2005) consider attributes of outcomes in a general framework, and Ortoleva (2010) (similar to Bewley (2002)) considers probability distributions over the state space.

According to the unanimity approach, once the criteria are determined, the decision maker is unwilling (or unable, depending on the interpretation) to perform any sort of compromise, with respect to this collection, relative to the status quo. However, decision makers may often allow for compromises in some relevant criteria while simultaneous improvements in other criteria (that are subjectively important enough) are guaranteed (see, for example, Soelberg (1967), Sheridan, Richards, and Slocum (1975) and Gensch (1987) from the Marketing literature). For instance, an assistant professor might take an associate professor position in a different state despite the uncertainty whether such a move will be successful or not, as long as she evaluates the chances of success to be sufficiently high.

Taking a choice-theoretic approach, the purpose of this paper is to present what we refer to as the *Probabilistic Dominance Model* of decision making under uncertainty: the decision maker considers an alternative choosable only if its chances of incurring a (subjectively) severe loss, relative to the status quo, are not too high.

1.1. **Main results.** In order to illustrate the Probabilistic Dominance Model, we adopt a setup similar to that in Masatlioglu and Ok (2005) (see also, Rubinstein and Zhou (1999), Sugden (2003), Munro and Sugden (2003), Sagi (2006), Salant and Rubinstein (2008), Ortoleva (2010) and Masatlioglu and Ok (2014)) but focus on inherent uncertainty. In this setup, a choice problem is either a collection of acts, or a collection of acts and a status quo (which is an act) in this collection. In particular, it is assumed that the status quo is always a feasible alternative. The decision maker is characterized by a choice correspondence that assigns a non-empty sub-collection of alternatives for each choice problem, with or without a status quo.

We present a set of axioms implying that the decision maker is associated with a subjective prior over the state space, where her choice is described as follows. Whenever there is no status quo, she simply acts as a standard subjective expected utility maximizer. However, if the decision problem is governed by a status quo, she resorts to a two-stage choice procedure. She first applies *probabilistic dominance* considerations to eliminate all alternatives that return an outcome significantly worse than the status quo with high enough probability. She then maximizes expected utility over the collection of feasible acts that survive the elimination stage (note that this collection is never empty, as the status quo always survives the elimination stage).

To illustrate the proposed model, consider for instance the assistant professor example mentioned above. When contemplating the different offers, the assistant professor compares each offer to the one from the university she is currently employed at, trying to assess the quality of life she and her family would experience both from professional and personal aspects. Out of all offers, she is willing to consider only those that guarantee, with (subjectively) sufficiently high probability, a quality of life not significantly worse than the one she would experience in case she accepted the offer from her current working place. Then, out off all the offers that survive this first stage, she chooses the one that maximizes their expected well-being.

Probabilistic dominance considerations describe to what extent the decision maker distinguishes between the status quo and new alternatives. Such considerations guarantee that the probability she will regret having moved away from the status quo (once uncertainty is resolved) is not too high. Alternatively, probabilistic dominance could be considered as ensuring some level of confidence when replacing the status quo; regardless of her choice, the chances the decision maker endures a (subjectively) severe loss relative to the status quo are not too high. For example, the assistant professor would like to convince her spouse (or alternatively, herself) that a particular offer is 'good enough' in the sense that the chance that it will improve their quality of life is significant.[1]

Another interpretation of the probabilistic dominance threshold arises when considering a framework of choices by committees. In such a case, each "state of the world" is considered to be a voter in a committee. Given such interpretation, each alternative, including the status quo (say, the current policy enforced by the company), is associated with an opinion profile. At first the committee votes on a short list—each member votes for the alternatives that he or she perceives better than the status quo, and only alternatives with sufficiently many votes are considered. Next, the committee proceeds with a more thoughtful discussion to decide on the final choice.

1.2. **Related literature.** Status quo bias was first captured and termed by Samuelson and Zeckhauser (1988). Through laboratory and field experiments, they indicated the strong affinity of individuals to retain the alternative which is the status quo. This observation has lead to a significant number of empirical studies on the presence of status quo bias (and related effects such as the endowment effect and reference-dependence) in important choices. Examples are 401(k) pension plans (Madrian and Shea (2001), Agnew, Balduzzi, and Sundén (2003) and Choi, Laibson, Madrian, and Metrick (2004)), electrical services (Hartman, Doane, and Woo (1991)) and car insurance (Johnson, Hershey, Meszaros, and Kunreuther (1993)).

These findings promoted the development of decision making models attempting to capture status quo effects. Kahneman and Tversky (1979) and (1991) presented a

[1]See the discussion in Shafir et al., (1993) page 33.

reference-dependent choice model based on loss aversion. The intuition behind this model is that the status quo affects the utility of the decision maker in such a manner that relative losses loom larger than corresponding gains. Sugden (2003) and Munro and Sugden (2003) axiomatize reference-dependent preferences in the spirit of Tversky and Kahneman (1991)'s 'loss-aversion'.

Masatlioglu and Ok (2005) and (2014) present a new approach to modeling status quo bias. Instead of affecting the underlying utility as in loss aversion, the presence of the status quo imposes mental constraints on what is choosable and what is not.[2] These constraints make some of the alternatives appear as inferior to the status quo and therefore unchoosable.[3] In Masatlioglu and Ok (2005), the status-quo bias axiom implies that the mental constraints admit a unanimity representation.[4]

Masatlioglu and Ok (2014) weaken the status-quo bias axiom and consider a much broader model. In particular, it is broader than the model presented here. For example, it is possible in the second stage of the process for the decision maker to consider some, *but not all*, alternatives that with high probability yield a good outcome relative to the status quo, as well as alternatives that yield a good outcome relative to the status quo with much lower probabilities.

Lastly, similarly to the current paper, Ortoleva (2010) studies the implications of status-quo bias in the presence of uncertainty. He adopts the Masatlioglu and Ok (2005) status-quo bias axiom and obtains a unanimity representation in the sense of Bewley (2002), where the criteria consist of different prior beliefs over the state space.

1.3. **Organization.** In the following section we discuss the main model in detail, describing the framework in Section 2.1, introducing the representation in Section 2.2, the axioms in Section 2.3 and the representation theorem in Section 2.4. Further discussion appears in Section 2.5. In Section 3 we present a natural notion of comparative bias

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[2]Rubinstein and Zhou (1999) consider a similar approach where the choosable alternatives are those closest to the status quo.

[3]See Masatlioglu and Ok (2014) for an in-depth discussion on the differences between the two approaches.

[4]Sagi (2006) studies the implications of an axiom bearing the essence of the Masatlioglu and Ok (2005) status-quo bias axioms. The example given by Sagi, when considered in the present framework, describes an attitude towards the status-quo which is identical to unanimity.

revealed towards the status quo and provide its implications to the probabilistic dominance model. Section 4 discusses two applications of the model. In Section 4.1, we use the model to investigate the willingness to pay and willingness to buy of an individual who suffers from the endowment effect. Section 4.2 discusses an application of the model to the problem of international portfolio choice. Lastly, all the proofs appear in the Appendix.

2. The main model

2.1. **The framework.** We follow the setup and notation in Masatlioglu and Ok (2005) with the difference that the objects of choice here are assumed to be "modern" Anscombe and Aumann acts. Formally, let X be a non–empty convex subset of a vector space, which we interpret as the *space of consequences*. Let S be a finite non–empty set of *states of nature*. Now, consider the collection $\mathcal{F} = X^S$ of all functions from states of nature to consequences. Such functions are referred to as *acts*. We denote by $\mathcal{F}_c$ the collection of all constant acts. Following the standard abuse of notation, we denote by x the constant act that assigns the consequence x to every state of nature. Similarly, given an act f and a state s, we write $f(s)$ to represent the constant act that returns the consequence $f(s)$ in every state of nature. For any two acts f and g and event $T \subseteq S$, we denote by fTg the act that assigns $f(s)$ to every state $s \in T$ and $g(s)$ to every state $s \in S \setminus T$. Lastly, mixtures (convex combinations) of acts are performed state-wise. For $f, g \in \mathcal{F}$ and $\alpha \in [0, 1]$, the act $\alpha f + (1 - \alpha)g$ returns $\alpha f(s) + (1 - \alpha)g(s)$, for each state $s \in S$. If A is a non-empty collection of acts, we write $\alpha A + (1 - \alpha)g$ to denote the collection $\{\alpha f + (1 - \alpha)g \ : \ f \in A\}$.

Let $\mathfrak{F}$ denote a collection of nonempty subsets of $\mathcal{F}$, such that all subsets of $\mathcal{F}$ with three or less elements belong to $\mathfrak{F}$.[5] The symbol $\diamond$ will be used to denote an object that does not belong to $\mathcal{F}$. By a *choice problem*, we mean a list (A, f) where $A \in \mathfrak{F}$ and either $f \in A$ or $f = \diamond$. The set of all choice problems is denoted by $\mathcal{C}(\mathcal{F})$. If $f \in A$, then the choice problem (A, f) is referred to as a *choice problem with a status quo*. We make

[5]This is an important assumption; without it the version of the Weak Axiom of Revealed Preference we use below loses part of its power. For example, $\mathfrak{F}$ may be the collection of all non-empty finite subsets of $\mathcal{F}$, or, in the case where there is a topology on X , $\mathfrak{F}$ may be the collection of all non-empty compact subsets of $\mathcal{F}$.

the explicit assumption that if there is a status quo, it must be feasible (note, however, that probabilistic dominance considerations can also be applied when the status quo is not feasible and serves only as a reference point). The interpretation is that the agent has to make a choice from the set A while the alternative f is her default option. We denote by $\mathcal{C}_{sq}(\mathcal{F})$ the set of all choice problems with a status quo. Finally, the notation $(A, \diamond)$, with $A \in \mathfrak{F}$ is used to represent a choice problem without a status quo.

The decision maker (henceforth, DM) is associated with a *choice correspondence*, that is, a map $c : \mathcal{C}(\mathcal{F}) \to 2^{\mathcal{F}} \setminus \{\emptyset\}$ such that

$$c(A, f) \subseteq A \text{ for all } (A, f) \in \mathcal{C}(\mathcal{F}).$$

2.2. **Representation.** We define a probabilistic dominance choice model as follows:

Definition 1. *A correspondence $c : \mathcal{C}(\mathcal{F}) \to 2^{\mathcal{F}} \setminus \{\emptyset\}$ is a* probabilistic dominance choice correspondence *if there exist an affine function $u : X \to \mathbb{R}$, a prior π over S, a $\theta \in [0, 1]$ and a scalar $\gamma \in \mathbb{R}_+$ such that, for all $A \in \mathfrak{F}$,*[6]

$$c(A, \diamond) = \arg\max_{f \in A} \; E_\pi \left[u(f) \right]$$

and, for all $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A, g) = \arg\max_{f \in \mathcal{D}(A,g,\pi,\theta,\gamma)} \; E_\pi \left[u(f) \right],$$

where, for each $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$\mathcal{D}\left(A, g, \pi, \theta, \gamma\right) := \{f \in A : \pi\{s : u(f(s)) \geq u(g(s)) - \gamma\} \geq \theta\}.$$

The interpretation of this choice procedure is the following. In the absence of a status quo, the agent acts as a standard subjective expected utility maximizer. As discussed in the Introduction, when faced with a decision problem governed by a status quo the agent wishes to be confident that, regardless of her choice, the chances she endures a (subjectively) severe loss, relative to the status quo, are not too high. She first eliminates all those alternatives that do not ensure such a sufficiently high level of confidence.

[6]**Notation:** By $E_\pi \left[u(f) \right]$ we mean the expected value of the random variable $u(f)$ with respect to the prior π.

Following the elimination stage, she acts as a standard expected utility maximizer as in choice problems without a status quo.[7]

The threshold parameters θ, γ capture the degree of confidence the DM wishes to ensure when moving away from the status quo. Fixing tastes (i.e., utility) and beliefs (i.e., prior distribution), it is intuitive that the larger the θ and smaller the γ, the more bias towards the status quo the DM is going to exhibit.(We make this intuition formal in Section 3.) When $\theta = 0$, the DM acts as a standard expected utility maximizer and exhibits no bias at all towards the status quo. At the other extreme, a DM who is characterized by $\theta = 1, \gamma = 0$ displays unanimity, and is not willing to take any chance of loosing by moving away from the status quo.[8]

2.3. **Axioms.** We impose the following properties on a choice correspondence c. We start by describing the more standard and familiar assumptions, and then move on to present the new properties.

A0 *Unboundedness.* For any $x, y \in X$ and $\alpha \in (0,1)$, there exist $w, z \in X$ such that $x \in c(\{x, \alpha y + (1-\alpha)w\}, \diamond)$ and $x \notin c(\{x, \alpha y + (1-\alpha)z\}, \diamond)$.

A1 *WARP.* If $(A,h), (B,h) \in \mathcal{C}(\mathcal{F})$ are such that $B \subseteq A$ and $c(A,h) \cap B \neq \emptyset$, then $c(B,h) = c(A,h) \cap B$.

A2 *Independence.* For any $f, g, h \in \mathcal{F}$ and $\alpha \in (0,1)$,

(1) $f \in c(\{f,g\},\diamond) \iff \alpha f + (1-\alpha)h \in c(\{\alpha f + (1-\alpha)h, \alpha g + (1-\alpha)h\}, \diamond)$ and
(2) $f \in c(\{f,g\},g) \implies \alpha f+(1-\alpha)h \in c(\{\alpha f+(1-\alpha)h, \alpha g+(1-\alpha)h\}, \alpha g+(1-\alpha)h)$.

[7] In a different framework and in a status-quo free context, sequentially rationalizable choices were also studied by Manzini and Mariotti (2007), Cherepanov, Feddersen, and Sandroni (2013) and Apesteguia and Ballester (2013). Two-staged decision processes are also of importance in the marketing literature (see Sheridan et al. (1975) and Gensch (1987) and references therein). Experimental and empirical evidence point to the fact that, when facing a choice problem, individuals tend to eliminate some alternatives according to a crude, non-compensatory initial rule, followed by a more thoughtful and compensatory process for deciding on the final choice.

[8] In the latter case, the states in the support of the subjective prior play the role of "criteria" (as discussed in the Introduction) under consideration.

A3 *Continuity.* For any $f, g, h \in \mathcal{F}$ and $j \in \{h, \diamond\}$, the set $\{\alpha : \alpha f + (1-\alpha)g \in c(\{\alpha f + (1-\alpha)g, h\}, j)\}$ is closed.

A4 *Dominance.* For any $f, g \in \mathcal{F}$ and $A \in \mathfrak{F}$ with $f(s) \in c(\{f(s), g(s)\}, \diamond)$ for every $s \in S$ and $\{f, g\} \subseteq A$, if $g \in c(A, h)$ for some $h \in A$, then $f \in c(A, h)$, and if $f \notin c(A, f)$, then $g \notin c(A, g)$.

A5 *Status quo Irrelevance (SQI).* For any $(A, f) \in \mathcal{C}_{sq}$, if there does not exist a non-singleton $B \subseteq A$ with $(B, f) \in \mathcal{C}_{sq}$ and $\{f\} = c(B, f)$, then $c(A, f) = c(A, \diamond)$.

The first assumption **A0** is an unboundedness assumption. This is essentially a technical condition we impose in order to guarantee that the utility function that will represent the individual's choices in the absence of a status quo is unbounded. The postulate **A1** is an adaptation of the standard Weak Axiom of Revealed Preference to the environment here. It says that if we keep the status quo (or the absence of status quo) fixed, then c satisfies that postulate. The first part of our independence axiom **A2** is also standard. It says that the agent's choices in status quo free problems with two alternatives satisfy the well-known independence postulate. The second part imposes only one of the two implications for problems with a status quo. The act f might be very different from the act g and the concern of such a substantial change may lead the agent to choose g over f when g is the status quo. However, when we mix f and g with an act h, with high enough weight on h, the change becomes less dramatic and it is conceivable that now the agent chooses $\alpha f + (1-\alpha)h$ over $\alpha g + (1-\alpha)h$ even when $\alpha g + (1-\alpha)h$ is the status quo. This is the reason why we impose only one of the implications of the Independence axiom in problems with a status quo.

Axiom **A3** is the known continuity condition, except for the fact that it allows for the presence of a status quo.

The dominance axiom **A4** imposes some consistency between the agent's choices in problems with and without a status quo when an act f is unambiguously better than another act g in the absence of a status quo. This property is reminiscent of the Unambiguous Transitivity axiom used by Lehrer and Teper (2011) and Faro (2014).

Axiom **A5** was first introduced by Masatlioglu and Ok (2014). It begins with a set A such that in no subset of A the bias towards the status quo f is strong enough to make it the only choice. In other words, the bias towards f is not significant, in the sense that the DM always considers moving away from it as something acceptable. The axiom then requires that such a default option does not affect the DM's behavior. That is, she resolves her problems as if there existed no status quo.[9]

Remark 1. *Axiom SQI can be weakened. All results in this paper are still true if we impose this property only for sets with three or less elements. We note that if we replace SQI by this simpler version then, except for Unboundedness and Continuity, our axiomatic system is entirely testable.*

For the next axiom, it will be helpful to present the following definition. Suppose that an act f is chosen in the presence of a status quo g. Assume now that $x\{s\}f$ is being chosen against $y\{s\}g$ in the status quo free problem, but no longer chosen when $y\{s\}g$ is the status quo. In a way, the comparison between x and y, where y has a reference status, was the factor for the agent's change of mind. This motivates the following:

Definition 2. *An alternative y* reference-dominates *another alternative x if there exist $f, g \in \mathcal{F}$ and $s \in S$ such that $f \in c(\{f, g\}, g)$, $x\{s\}f \in c(\{x\{s\}f, y\{s\}g\}, \diamond)$, but $x\{s\}f \notin c(\{x\{s\}f, y\{s\}g\}, y\{s\}g)$.*

We are now ready to state the following postulate:

A6 *Reference Dominance Coherence.* If the alternative y reference-dominates the alternative x, then, for any $f, g \in \mathcal{F}$ and $s \in S$, $f \in c(\{f, g\}, \diamond)$ and $f \notin c(\{f, g\}, g)$ implies $x\{s\}f \notin c(\{x\{s\}f, y\{s\}g\}, y\{s\}g)$.

In the postulate above, we have a pair of alternatives where y has already been revealed to reference-dominate x. The postulate then asks that whenever an act f is not chosen against another act g because g is the status quo, it cannot happen that $x\{s\}f$ is more

[9]There are two forces behind **A5**. First, there is the idea that a weak status quo should not affect the agent's choices. The following postulate is a description of this idea: for any $(A, f) \in \mathcal{C}_{sq}$, if there does not exist a non-singleton $B \subseteq A$ with $(B, f) \in \mathcal{C}_{sq}$ and $f \in c(B, f)$, then $c(A, f) = c(A, \diamond)$. The other force behind **A5** is the idea that in the presence of a status quo, the individual's decision procedure has two stages and the second stage agrees with her choices when there is no status quo. This is captured by the following property: for any $f, g \in \mathcal{F}$, if $\{f, g\} = c(\{f, g\}, f)$, then $\{f, g\} = c(\{f, g\}, \diamond)$. When c satisfies WARP, the two properties above together are equivalent to **A5**.

appealing than $y\{s\}g$, when $y\{s\}g$ itself is the status quo. In other words, an alternative never has a positive effect when compared with an alternative that reference-dominates it.

A7 *Binary Consistency.* Let $x, y, z \in X$. If f and g are acts such that $f(S) \cup g(S) \subseteq \{x, z\}$, then $\{f\} = c(\{f, g, y\}, y)$ implies that $\{f\} = c(\{f, g, y\}, \diamond)$.

To describe **A7** in detail, define a relation $\succcurlyeq$ over all acts by $f' \succcurlyeq g'$ if and only if $f' \in c(\{f', g'\}, \diamond)$. It is easily checked that $\succcurlyeq$ is a complete preorder. Now suppose that f and g are acts that return only x and z. Without loss of generality, assume that $x \succcurlyeq z$. We can show that, unless $x \succ y \succ z$, Binary Consistency is implied by the other axioms presented above.[10] So, the actual content of Binary Consistency comes from the case $x \succ y \succ z$, which we now assume. Let $T := \{s \in S : f(s) = x\}$ and $\hat{T} := \{s \in S : g(s) = x\}$. The acts f and g can be interpreted as bets on the events T and $\hat{T}$, respectively. The axiom then postulates a consistency property, imposing that if the DM prefers the bet on the event T in the presence of the status quo alternative y, then she should also prefer this bet in the absence of a status quo.

2.4. **The representation theorem.** Our main result is the following representation theorem.

Theorem 1. *Given a choice correspondence $c : \mathcal{C}(\mathcal{F}) \to 2^{\mathcal{F}} \setminus \{\emptyset\}$ the following are equivalent:*
*(1) c satisfies **A0-A7**;*
(2) c is a probabilistic dominance choice correspondence with a representation (u, π, θ, γ) such that $u(X) = \mathbb{R}$.

We would like to formally state a uniqueness result for the parameters representing a probabilistic dominance choice correspondence. Following the proof of Theorem 1, it is clear that there is no unique θ that represents c; furthermore, θ can be chosen from some interval. Consider, for example, a state space with two states and a probabilistic dominance choice correspondence that is represented by a non-constant utility, a prior

[10]**Proof:** If $y \succcurlyeq x$, then Dominance implies that $y \in c(\{f, g, y\}, y)$ and, consequently, $\{f\} \neq c(\{f, g, y\}, y)$. If $z \succcurlyeq y$, then Dominance implies that $f \in c(\{f, y\}, y)$ and $g \in c(\{g, y\}, y)$. Now SQI implies that $c(\{f, g, y\}, y) = c(\{f, g, y\}, \diamond)$.

distribution assigning probability 0.5 for each state and $\gamma = 0$. Holding these parameters fixed, one would obtain identical choices for every $0 \le \theta \le 0.5$. For each such θ, in every choice problem the corresponding procedure would simply pick those alternatives with highest expected utility. Similarly, for every $0.5 < \theta \le 1$ the choice procedure is identical; the decision maker maximizes expected utility only over those alternatives that (state-wise) dominate the status quo. In this example, even though different θ's represent the same choices, they all have a common feature: the collection of all events with probability at least θ coincides for all θ's that induce the same choices.

In order to formalize this observation, let (u, π, θ, γ) be a representation of a probabilistic dominance choice correspondence as in Theorem 1. Now, define $\mathcal{T}_\theta := \{T \in 2^S \setminus \{\emptyset\} : \pi(T) \ge \theta\}$. The collection $\mathcal{T}_\theta$ is *decisive* in the sense that if an alternative does not dominate the status quo over at least one of the events in $\mathcal{T}_\theta$, it is eliminated and is implicitly excluded from the feasible set by the DM. Having this in mind, we can present the following uniqueness result.

Proposition 1. *Let c be a choice correspondence such that $c(A, \diamond) \neq c(A, g)$ for some $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$. If (u, π, θ, γ) and $(\hat{u}, \hat{\pi}, \hat{\theta}, \hat{\gamma})$ are two representations of c as described in Theorem 1, then $\hat{\pi} = \pi$, $\mathcal{T}_\theta = \mathcal{T}_{\hat{\theta}}$ and there exist $\alpha \in \mathbb{R}_{++}$ and $\beta \in \mathbb{R}$ such that $\hat{u} = \alpha u + \beta$ and $\hat{\gamma} = \alpha\gamma$.*

The uniqueness of u and π is simply the uniqueness of subjective expected utility representations. If there exists at least one situation where the presence of a status quo option changes the DM's choices, then we also get the uniqueness of the parameter γ. The collection of decisive events $\mathcal{T}_\theta$ is also unique. If the DM's choices in the presence of a status quo always agree with her choices in the absence of a status quo, then one can only obtain uniqueness of the expected utility representation and of the collection $\mathcal{T}_\theta$. In this case, $\mathcal{T}_\theta = 2^S \setminus \{\emptyset\}$ and we can use any $\gamma \ge 0$ to represent c.

It is interesting to see the relation, at least intuitively, between the axioms, the representation, and related existing models. The formal details appear in the proof of the Theorem 1. Axioms **A1** and **A5** were applied jointly in Masatlioglu and Ok (2014), resulting in the following. Status quo free choices follow (classic) utility maximization. Choices in the presence of a status quo follow a two stages procedure. In the first stage the decision maker eliminates alternatives according to some abstract criterion, invariant over all problems with the same status quo. She then maximizes the same utility as in the status quo free problems, but only over the alternatives that survived the first stage. Axioms **A2**, **A3**, and **A4** ensure that the utility the decision maker maximizes is the expected utility (as in Anscombe and Aumann (1963)) according to a prior belief over the state space. Axioms **A6** and **A7** are novel. **A6** implies that elimination of alternatives in the presence of a status quo is more structured; there exists a collection of decisive events $\mathcal{T}$ and during the first stage of the decision process, an act has to perform in a non-disastrous manner relative to the status quo over at least one decisive event. However, $\mathcal{T}$ need not be consistent with the agent's subjective belief π and there might exist some event $T \in \mathcal{T}$ and an event $T' \notin \mathcal{T}$ such that $\pi(T) < \pi(T')$. Axiom **A7** implies that such inconsistencies do not occur and that there exists a probabilistic threshold that characterizes the decisive events.

Lastly, a natural and interesting particular case of probabilistic dominance choice is the one in which $\gamma = 0$. Such a model can be obtained by simply making the second part of our independence axiom **A2** also an "if and only if" property. That is, the second part of **A2** becomes $f \in c(\{f,g\},g) \iff \alpha f + (1-\alpha)h \in c(\{\alpha f + (1-\alpha)h, \alpha g + (1-\alpha)h\}, \alpha g + (1-\alpha)h)$. Formulated for preference relations, a similar version of this axiom appeared in Ortoleva (2010).

2.5. **Discussion.**

2.5.1. *Infinite state space and Savagean framework.* The analysis in the paper is done for a finite state space. However, a general state space can be considered as well. In that case **A6** would have to be written in a somewhat stronger fashion, since a single state no longer has any meaningful "contribution" to the assessment of an act. In this case, the reformulated axiom would take into account variations over events.

The current paper adopts the Anscombe and Aumann setup. This is done mostly for the sake of tractability, but the convex structure of the prize space in the Anscombe and Aumann framework can also be useful in applications (see Section 4, for example). The probabilistic dominance choice model can be formulated in a Savagean framework. However, there is a price to pay due to the axiomatic structure in such a setting. Like in Savage (1954), one would have to reformulate our independence Axiom **A2**.

2.5.2. *Continuity.* According to our model, an act that is essentially the same as another in most states and much better in some others can be rejected. The uneasiness of the example above is a consequence of the fact that models of sequential decision making induce discontinuous choice correspondences. Because most of the sequential choice literature is developed in a setup with a finite space of alternatives, this issue is usually not evident. Here, the Ancombe-Aumann framework provides a natural measure of how similar two acts are and makes the problem more salient. Although such "similarities" may not exist in a Savage (1954) framework, it is possible to generate discontinuities for sequential choice models in such a setting as well, due to the non-atomicity of the subjective prior.

We view the behavior described by the model introduced here as an approximation and consider such consequences as a price we have to pay to obtain a tractable model. Nevertheless, discontinuities similar to those that occur with the model presented here, may occur in practice. One example of that is the use of the value at risk measure as a regulatory device for managers of trusts and pension plans. In such situations, managers operate with the additional constraint of having to avoid portfolios that can incur a big loss with a probability above a certain threshold. This procedure is very much in the spirit of our model and exhibits the same type of discontinuities.

3. Comparative status quo bias

Consider two DMs, I and II, associated with choice correspondences c_1 and c_2, respectively. We say that c_1 reveals more bias towards the status quo than c_2 if whenever c_2 reveals status quo bias then so does c_1. Formally:

Definition 3. c_1 reveals more bias towards the status quo *than* c_2 *if, for every* $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, $\{g\} = c_2(A, g)$ *implies* $\{g\} = c_1(A, g)$.

The following proposition reinforces the intuition behind our interpretation of probabilistic dominance choice and, in particular, the formation of decisive events.

Proposition 2. *Suppose* c_1 *and* c_2 *are two probabilistic dominance choice correspondences (as axiomatized in Theorem 1). Let* $(u_i, \pi_i, \theta_i, \gamma_i)$ *be the corresponding representations for* c_1 *and* c_2*, and assume that* $\gamma_1 > 0$*. Then, the following are equivalent:*
(1) c_1 *reveals more bias towards the status quo than* c_2*;*

(2) there exist $\alpha \in \mathbb{R}_{++}$ *and* $\beta \in \mathbb{R}$ *such that* $u_2 = \alpha u_1 + \beta$, $\pi_1 = \pi_2$ *and* $(u_2, \pi_2, \min(\theta_1, \theta_2), \max(\alpha\gamma_1, \gamma_2))$ *represents* c_2.

What the proposition suggests is simply that revealing less status quo bias is equivalent to: 1. having a smaller probabilistic threshold parameter θ; 2. allowing for potentially larger losses relative to the status quo. Because of the non-uniqueness of the probabilistic parameter threshold θ, we cannot exactly say that $\theta_1 \geq \theta_2$, so statement (2) says instead that it is always possible to represent c_2 with a θ_2 not greater than θ_1. In particular, statement (2) implies that $\mathcal{T}_{\theta_1} \subseteq \mathcal{T}_{\theta_2}$, where $\mathcal{T}_{\theta_1}$ and $\mathcal{T}_{\theta_2}$ are the collection of decisive events induced by θ_1 and θ_2, respectively. Note also, that because of the uniqueness result presented in Proposition 1, statement (2) implies that whenever c_2 is not simply a subjective expected-utility maximizer we will necessarily have that $\alpha\gamma_1 \leq \gamma_2$.

Remark 2. *If we do not assume that* c_1 *is represented with a* $\gamma_1 > 0$*, similar results to Proposition 2 can still be obtained, but in this case we have to impose that* $c_1(., \diamond) = c_2(., \diamond)$*, as this is no longer implied by the definition of revealing more bias towards the status quo.*

4. Applications

4.1. **The endowment effect.** People often demand more in order to give up a commodity than they would be willing to pay to acquire it (see Kahneman, Knetsch, and Thaler (1991) for a survey). This phenomenon, intrinsically related to status quo bias, is the well-documented *endowment effect* (Thaler (1980)). We show that our model predicts this phenomenon in terms of unambiguous gains of utility. Also, we provide a simple way to compute, in terms of utility, the willingness to accept (WTA) and willingness to pay (WTP).

Consider a probabilistic dominance choice correspondence c that has a representation with a non-constant and affine function $u : X \to \mathbb{R}$, a prior π over S and threshold parameters $\theta \in [0, 1]$ and $\gamma \geq 0$. We define the function $S_c : \mathcal{F} \to \mathbb{R}$ by[11]

$$S_c(f) := \inf\{u(x) : x \in X \text{ and } x \in c(\{x, f\}, f)\}$$

[11] Similar definitions, in an environment with no uncertainty and explicit potential prices, can be found in Masatlioglu and Ok (2005).

and the function $B_c : \mathcal{F} \to u(X)$ by

$$B_c(f) := \sup\{u(x) : x \in X \text{ and } f \in c(\{x, f\}, x)\}.$$

Intuitively, $S_c(f)$ is the minimum unambiguous gain the individual would require in order to give away the act f (WTA in terms of utility) and $B_c(f)$ is the maximum unambiguous gain the individual would be willing to give away in order to have f (WTP in terms of utility).

For a constant act x to be chosen from the pair $\{f, x\}$ when f is the status quo, two conditions must be fulfilled. Its utility (vector) must dominate $u(f) - \gamma$ with probability at least θ, and its utility must be as high as the expected utility induced by f. Thus, given the representation of c, we have that

$$S_c(f) = \max\left\{E_\pi\left[u(f)\right], \min\{v \in \mathbb{R} : \pi\{s : u(f(s)) - \gamma \leq v\} \geq \theta\}\right\}. \tag{1}$$

Applying similar arguments as above (for the case where the constant act x is the status quo when choosing between f and x), we obtain

$$B_c(f) = \min\left\{E_\pi\left[u(f)\right], \max\{v \in \mathbb{R} : \pi\{s : u(f(s)) + \gamma \geq v\} \geq \theta\}\right\}. \tag{2}$$

Loosely speaking, the two terms above, $\min\{v \in u(X) : \pi\{s : u(f(s)) - \gamma \leq v\} \geq \theta\}$ and $\max\{v \in u(X) : \pi\{s : u(f(s)) + \gamma \geq v\} \geq \theta\}$, are expressions for the left θ quantile and the right $1-\theta$ quantile of the distributions of $u(f)-\gamma$ and $u(f)+\gamma$ with respect to π, respectively. We denote them $\widetilde{Q}_{\pi,\theta}(f)$ and $\widehat{Q}_{\pi,1-\theta}(f)$, respectively. Thus, the WTA and WTP of any act can be calculated using only its expected value and these two quantiles.

When c can be represented with $\theta = 0$ and the DM is not status quo biased, choosing as a standard expected utility maximizer, for every act f we have that $\widehat{Q}_{\pi,1}(f) \geq \mathbb{E}_\pi\left[u(f)\right] \geq \widetilde{Q}_{\pi,0}(f)$, meaning that $B_c(f) = S_c(f)$. The question is how $B_c(f)$ and $S_c(f)$ relate whenever c reveals a bias towards the status quo and cannot be represented with $\theta = 0$.

From Eq. (1) and (2) it is easy to verify that, for a particular act f, the WTA is at least as high as the WTP. Also, a necessary and sufficient condition for both to be equal is $\widehat{Q}_{\pi,1-\theta}(f) \geq \mathbb{E}_\pi\left[u(f)\right] \geq \widetilde{Q}_{\pi,\theta}(f)$. Furthermore, if c cannot be represented with $\theta = 0$, there always exists an act f for which the latter inequalities do not hold. This implies that the WTA for this specific act is strictly higher than the WTP. We formalize these results in the next proposition:

Proposition 3. *Suppose that u, π, θ and γ represent a non-trivial probabilistic dominance choice correspondence c as in Theorem 1. Then,*
(a) $S_c(f) \geq B_c(f)$ for every $f \in \mathcal{F}$.
(b) For every $f \in \mathcal{F}$, $S_c(f) = B_c(f)$ if and only if $\widehat{Q}_{\pi,1-\theta}(f) \geq E_\pi\left[u(f)\right] \geq \widetilde{Q}_{\pi,\theta}(f)$.
(c) The following statements are equivalent:
  (1) There exists $f \in \mathcal{F}$ with $S_c(f) > B_c(f)$;
  (2) There exists $s^* \in S$ such that $\theta > \pi(s^*) > 0$; and
  (3) $\theta = 0$ does not represent c.

4.2. **Probabilistic dominance, portfolio choice and the home bias phenomenon.** It has been observed before that the home bias phenomenon may be seen as an instance of a reference dependent choice, where, of course, the reference point is to invest on one's home country. We apply this idea to investigate the portfolio choice of a home biased investor resorting to probabilistic dominance considerations. We show that in this setup our model presented here has interesting properties and that it is also analytically convenient.[12] For example, the amount the home biased investor invests abroad varies positively with a measure of how aligned the risks of the home and the international assets are (see Section 4.2.2). That is, the more aligned the risks in the home and the foreign country are, the more the investor invests abroad. Another property of the model is that the investor chooses her portfolio in two stages (see Section 4.2.3). First she chooses the best composition for the international part of her portfolio and, after that, she chooses how much to invest abroad. Finally, in Section 4.2.4, we show that the model has a nice aggregation property.

Our analysis concentrates on down-side risk averse investors as in Arzac and Bawa (1977). We show that the main results in Arzac and Bawa still hold when the status quo is not a secure act.

[12]Recently, increasing attention has been paid to implications of reference dependent choice models in setups of general equilibrium. Solnik and Zuo (2012), for example, performed such an exercise applying the standard regret theory à la Bell (1982) and Loomes and Sugden (1982). Similarly, motivated by the concept of familiarity bias, Cao, Han, Hirshleifer, and Zhang (2011) introduced a model where investors were status quo biased à la Bewley (2002) and comparisons between two non status quo options were made according to a procedure similar to the one introduced in Gilboa and Schmeidler (1989).

4.2.1. *Portfolio choice.* Consider a world with monetary acts. That is, an act is a function $f : S \to \mathbb{R}$. Acts, then, are *assets* the agent can buy. Each asset f has an initial market value V_f and the individual's initial wealth is denoted by W. The agent can buy any amount β_j of a set of $J+1$ assets, where $J \in \mathbb{N}$. Finally, we assume that the agent's choice procedure can be represented as in the previous sections with the status quo being to invest all the wealth W on an asset h, which we interpret as being the home asset. The problem of the investor can be written as

$$\max_{\beta_h, \{\beta_j\}_{j=1}^J} \sum_{j=1}^{J} \beta_j E[f_j] + \beta_h E[h]$$

s.t.

$$\beta_h \geq 0, \beta_j \geq 0 \text{ for } j = 1, ..., J, \tag{3}$$

$$\sum_{j=1}^{J} \beta_j V_j + \beta_h V_h = W,$$

$$P\left\{ \sum_{j=1}^{J} \beta_j f_j + \beta_h h - \frac{W}{V_h} h \geq -\gamma \right\} \geq \theta,$$

where $\theta \in (0, 1)$ and $\gamma > 0$.

In words, the agent chooses how much to buy of each asset in order to maximize the expected value of her portfolio. The first two constraints are standard. The first means that we are not allowing for shortsales.[13] The second is simply the individual's budget constraint. The last constraint is related to the fact that we assume that the agent's choice procedure can be represented as in Theorem 1, with the status quo being to invest all the wealth on the home asset. The constraint then says that the probability that the agent's portfolio, the act $\sum_{j=1}^{J} \beta_j f_j + \beta_h h$, represents a loss greater than γ when compared to the status quo, the act $\frac{W}{V_h} h$, has to be smaller than $1 - \theta$.

4.2.2. *Two assets.* Let's consider the simplified case where the investor's decision is to choose how much to invest abroad. That is, suppose that there are only two assets: the

[13] All results in this section are still true if we allow for that, but the algebra is more tedious and the statements of Assumptions 1 and 2 below become more technical.

home asset h and a foreign asset f, with investing everything on h being the status quo. In this case the investor's problem can be written as

$$\max_{\beta_f,\beta_h} \beta_f E\left[f\right] + \beta_h E[h]$$

s.t.

$$\beta_h, \beta_f \geq 0$$

$$\beta_f V_f + \beta_h V_h = W$$

$$P\left\{\beta_f f + \beta_h h - \frac{W}{Vh} h \geq -\gamma\right\} \geq \theta.$$

We make the assumption that the foreign asset has an expected return higher than the home asset, but investing everything on the foreign asset does not satisfy the third restriction.

Assumption 1. $\frac{E[f]}{V_f} > \frac{E[h]}{V_h}$ *and* $P\left\{\frac{W}{V_f} f - \frac{W}{V_h} h \geq -\gamma\right\} < \theta.$

Given a random variable x, define $q_{1-\theta}\left(x\right) := \max\left\{q \in \mathbb{R} : P\left\{x < q\right\} \leq 1-\theta\right\}$.[14] That is, $q_{1-\theta}\left(x\right)$ is the largest $1-\theta$ quantile of the cumulative distribution of x. We can now state the following proposition:

Proposition 4. *The optimal portfolio invests positive amounts on both assets and the amount it invests on the foreign asset is given by* $\beta_f = \frac{-\gamma}{V_f q_{1-\theta}\left(\frac{f}{V_f} - \frac{h}{V_h}\right)}$.

As a consequence of Assumption 1, $q_{1-\theta}\left(\frac{f}{V_f} - \frac{h}{V_h}\right) < \frac{-\gamma}{W} < 0$, so, the amount the individual invests on the foreign asset varies positively with the tolerance parameter γ. Also, $q_{1-\theta}\left(\frac{f}{V_f} - \frac{h}{V_h}\right)$ can be seen as a measure of how aligned the returns of the international and domestic assets are. We note that, differently from the variance of $\frac{f}{V_f} -$

[14]We note that, as long as there exists some $q \in \mathbb{R}$ with $P\left\{x < q\right\} \leq 1-\theta$, $q_{1-\theta}$ is well defined. To see that, let $q_{1-\theta} := \sup\left\{q \in \mathbb{R} : P\left\{x < q\right\} \leq 1-\theta\right\}$ and consider an increasing sequence $q^m \uparrow q_{1-\theta}$. Now note that,

$$\begin{aligned} P\left\{x < q_{1-\theta}\right\} &= P\left(\bigcup_{m=1}^{\infty}\left\{x < q^m\right\}\right) \\ &= \lim_{m\to\infty} P\left\{x < q^m\right\} \\ &\leq 1-\theta. \end{aligned}$$

$\frac{h}{V_h}$, for example, it is a measure that only takes into account the part of the distribution of $\frac{f}{V_f} - \frac{h}{V_h}$ where $\frac{f}{V_f} - \frac{h}{V_h} < \frac{-\gamma}{W}$.

4.2.3. *International portfolio.* Suppose now that there exist many international assets. That is, suppose that the agent's problem is written in the more general format introduced in Section 4.2.1. We make the assumption that the optimum portfolio will include a positive amount of the home asset and a positive amount of at least one international asset.

Assumption 2. *There exists an international asset f^* with $\frac{E[f^*]}{V_{f^*}} > \frac{E[h]}{V_h}$. Moreover, for any portfolio composed only of international assets that satisfies the budget constraint we have $P\left\{\sum_{j=1}^{J} \beta_j f_j - \frac{W}{V_h} h \geq -\gamma\right\} < \theta$.*

Now, define the return on the international part of the investor's portfolio by $R := \frac{\sum_{j=1}^{J} \beta_j f_j}{\sum_{j=1}^{J} \beta_j V_j}$, and the return on the home asset by $R_h := \frac{h}{V_h}$. Also, given an international portfolio $(\beta_1, ..., \beta_j)$, define, for $j = 1, ..., J$, $b_j := \frac{\beta_j}{\sum_{j=1}^{J} \beta_j}$. That is, for each j, b_j is the relative share of the asset j in the investors international portfolio. Note that R can be equivalently written as $R = \frac{\sum_{j=1}^{J} b_j f_j}{\sum_{j=1}^{J} b_j V_j}$. We can now state the following proposition:

Proposition 5. *A portfolio $(\beta_1, ..., \beta_j, \beta_h)$ is optimal if, and only if,*

(i): *The relative shares $(b_1, ..., b_j)$ induced by the international part of the portfolio solve the following problem:*

$$\max_{\{b_j\}_{j=1}^{J}} -\frac{E[R] - E[R_h]}{q_{1-\theta}\left(R - R_h\right)}$$

s.t.

$$b_j \geq 0 \text{ for } j = 1, ..., J \text{ and } \sum_{j=1}^{J} b_j = 1.$$

(ii): *The total monetary amount invested on international assets is given by*

$$\sum_{j=1}^{j} \beta_j V_j = \frac{-\gamma}{q_{1-\theta}\left(R - R_h\right)}.$$

Proposition 5 shows that the investor's problem can be solved in two stages. First, the investor chooses the best relative composition for her international portfolio by solving the problem in (i). After that, she can compute how much to invest abroad using the same expression we have found in the previous section.

We note that Proposition 5 implies that the composition of the investor's international portfolio depends only on the investor's beliefs and the probability parameter θ. The investor's initial wealth W and her acceptable loss parameter γ affect only how much she invests abroad, but they do not change the relative weights of the assets in her international portfolio. In other words, any two investors that share the same beliefs and the same parameter θ consider optimal the same international portfolios, differing only on the amount invested on them. In fact, there are cases where this holds even when the investors have different θ's. Suppose, for example, that $q_{1-\theta}(R-R_h)$ can be written in the format $q_{1-\theta}(R-R_h) = E[R-R_h] - \rho(1-\theta)\beta(R-R_h)$, where ρ and β are real valued functions such that $\rho, \beta > 0$. This is the case, for example, if $R-R_h$ is normally distributed and all investor's have θ's strictly greater than $1/2$.[15] In this case, maximizing $-\frac{E[R]-E[R_h]}{q_{1-\theta}(R-R_h)}$ is the same as maximizing $-\frac{E[R]-E[R_h]}{\beta(R-R_h)}$ and, once more, all investor's will agree on the optimal international portfolios.

4.2.4. *Aggregation and representative investor.* Let's now suppose we have a group of I investors who share the same beliefs and the same parameter θ. Let W_i be the initial wealth of investor i. Similarly, let γ_i be investor i's parameter γ in her representation. Also, suppose the problem in (i) of Proposition 3 has a unique solution. From what we have learned in the previous section, all investor's will choose international portfolios with the same relative weights. Let R be the random variable that represents the returns of the chosen international portfolio. We know that each investor i will invest an amount

$$\beta_{i,h} = \frac{1}{V_h}\left(W_i + \frac{\gamma_i}{q_{1-\theta}(R-R_h)}\right)$$

on the home asset. Summing up the expression above for all investors we get that the total investment on the home asset will be given by

$$\left(\sum_{i=1}^{I}\beta_{i,h}\right) = \frac{1}{V_h}\left(\sum_{i=1}^{I}W_i + \frac{\sum_{i=1}^{I}\gamma_i}{q_{1-\theta}(R-R_h)}\right).$$

We note that the total investment on the home asset is the same amount a single investor with wealth $\sum_{i=1}^{I}W_i$ and parameter $\sum_{i=1}^{I}\gamma_i$ in her representation would invest. That is, the model developed here has a nice aggregation property. As long as investor's

[15]In the case of the normal distribution, $\rho(1-\theta) := q_{1-\theta}(N(0,1))$, where $q_{1-\theta}(N(0,1))$ is the $1-\theta$ quantile of the normal distribution with mean zero and standard deviation one, and $\beta(R-R_h)$ is the standard deviation of $R-R_h$.

share beliefs and parameter θ, they behave as a single investor with the same kind of representation (see also Arzac and Bawa (1977)).

Appendix A. Proofs

A.1. **Proof of Theorem 1.** It is routine to show that the representation implies the axioms, so we only show that the axioms are sufficient for the representation. Define the relation $\succcurlyeq \subseteq \mathcal{F} \times \mathcal{F}$ by $f \succcurlyeq g$ iff $f \in c(\{f, g\}, \diamond)$. By WARP, $\succcurlyeq$ is a complete preorder and, for any $A \in \mathfrak{F}$, $c(A, \diamond) = \arg\max(A, \succcurlyeq)$.[16] By the first part of our independence axiom, $\succcurlyeq$ satisfies the standard independence axiom.[17] Continuity implies that for all $f, g, h \in \mathcal{F}$ the set $\{\alpha \in [0,1] : \alpha f + (1-\alpha) g \succcurlyeq h\}$ is closed. We need the following claim:

Claim 1. *For all $f, g, h \in \mathcal{F}$, the set $\{\alpha \in [0,1] : h \succcurlyeq \alpha f + (1-\alpha) g\}$ is closed.*

Proof of Claim. Let $\{\alpha^m\} \subseteq [0,1]$ be a convergent sequence such that $h \succcurlyeq \alpha^m f + (1 - \alpha^m) g$ for all m. Let $\bar{\alpha} := \lim \alpha^m$. Without loss of generality, we can assume that (α^m) is increasing and that $\alpha^m > 0$ for all m .[18] Now note that, by Independence, for all m,

$$h \succcurlyeq \alpha^m f + (1-\alpha^m) g \iff \frac{\alpha_1}{\alpha^m} h + (1 - \frac{\alpha_1}{\alpha^m}) g \succcurlyeq \alpha_1 f + (1-\alpha_1) g.$$

Applying our Continuity axiom we get that $\frac{\alpha_1}{\bar{\alpha}} h + (1 - \frac{\alpha_1}{\bar{\alpha}}) g \succcurlyeq \alpha_1 f + (1-\alpha_1) g = \frac{\alpha_1}{\bar{\alpha}} (\bar{\alpha} f + (1-\bar{\alpha}) g) + (1 - \frac{\alpha_1}{\bar{\alpha}}) g$. Applying Independence one more time we get the desired conclusion. $\square$

Now suppose that $f(s) \succcurlyeq g(s)$ for all $s \in S$. By Dominance, it must be the case that $f \in c(\{f, g\}, g)$. By SQI, we must have $f \in c(\{f, g\}, \diamond)$, which is equivalent to saying that $f \succcurlyeq g$. That is, $\succcurlyeq$ satisfies standard Monotonicity, in the sense that $f(s) \succcurlyeq g(s)$ for all $s \in S$ implies that $f \succcurlyeq g$. We have just shown that $\succcurlyeq$ satisfies all conditions for an Anscombe and Aumann representation. So, there exists an affine function $u : X \to \mathbb{R}$ and a prior π over S such that, for any $A \in \mathfrak{F}$,

$$c(A, \diamond) = \underset{f \in A}{\arg\max} E_\pi [u(f)]. \tag{4}$$

[16]**Notation:** By $\arg\max(A, \succcurlyeq)$ we mean the set $\{f \in A : f \succcurlyeq g \text{ for all } g \in A\}$.

[17]That is, for any acts $f, g, h \in \mathcal{F}$ and $\alpha \in (0,1)$, $f \succcurlyeq g \iff \alpha f + (1-\alpha) h \succcurlyeq \alpha g + (1-\alpha) h$.

[18]Take any monotone subsequence of (α^m) and, if necessary, reverse the wholes of α^m and $(1 - \alpha^m)$, and f and g.

We need the following claim:

Claim 2. *The function u satisfies $u(X) = \mathbb{R}$.*

Proof of Claim. By Unboundedness, it is clear that there exist $x, y \in X$ with $u(x) > u(y)$. Now fix any $k > \max\{|u(x)|, |u(y)|\}$ and let $\lambda := \frac{u(y)+k}{u(x)+k}$. By Unboundedness, there exists $w \in X$ such that $u(y) \geq \lambda u(x) + (1-\lambda)u(w)$. This can happen only if $u(w) \leq -k$. Now define $\lambda := \frac{k-u(x)}{k-u(y)}$. By Unboundedness, there exists $z \in X$ such that $u(x) < \lambda u(y) + (1-\lambda)u(z)$. This now implies that $u(z) > k$. This shows that u is unbounded above and below. Since u is affine, it must be the case that $u(X) = \mathbb{R}$. $\square$

Now define the relation $\succcurlyeq^* \subseteq \mathcal{F} \times \mathcal{F}$ by $f \succcurlyeq^* g$ iff $f \in c(\{f, g\}, g)$. We note that $\succcurlyeq^*$ is a reflexive binary relation. For each $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, define $\mathcal{D}((A, g), \succcurlyeq^*) := \{f \in A : f \succcurlyeq^* g\}$. We need the following claim:

Claim 3. *For every $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,*

$$c(A, g) = \underset{f \in \mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} E_\pi [u(f)].$$

Proof of Claim. By WARP, $f \in c(A, g)$ implies that $f \in c(\{f, g\}, g)$. This shows that $c(A, g) \subseteq \mathcal{D}((A, g), \succcurlyeq^*)$. Now pick any $(f, h) \in c(A, g) \times \mathcal{D}((A, g), \succcurlyeq^*)$. If $f = h$, then it is clear that $E_\pi[u(f)] \geq E_\pi[u(h)]$. Suppose, then, that $f \neq h$. By WARP, $f \in c(\{f, g, h\}, g)$, and, by SQI, $c(\{f, g, h\}, g) = c(\{f, g, h\}, \diamond)$. By (4), this implies that $E_\pi [u(f)] \geq E_\pi [u(h)]$. We conclude that $c(A, g) \subseteq \underset{f \in \mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} E_\pi [u(f)]$. In particular, this shows that $\underset{f \in \mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} E_\pi [u(f)] \neq \emptyset$. Now pick $h \in \underset{f \in \mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} E_\pi [u(f)]$ and $f \in c(A, g)$. Note that this implies that $E_\pi [u(h)] \geq E_\pi [u(f)]$ and $E_\pi [u(h)] \geq E_\pi [u(g)]$. Again, if $f = h$, then it is clear that $h \in c(A, g)$. Suppose, then that $f \neq h$. By SQI, we know that $c(\{f, g, h\}, g) = c(\{f, g, h\}, \diamond)$, which, by (4), implies that $h \in c(\{f, g, h\}, g)$. But, by WARP, $c(\{f, g, h\}, g) = c(A, g) \cap \{f, g, h\}$. We conclude that $\underset{f \in \mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} E_\pi [u(f)] \subseteq c(A)$. $\square$

We note now that $\succcurlyeq^*$ satisfies the following property:

Claim 4. *For any $f, g, h, j \in \mathcal{F}$, if $u(f(s)) - u(g(s)) \geq u(h(s)) - u(j(s))$ for every $s \in S$, and $h \succcurlyeq^* j$, then $f \succcurlyeq^* g$.*

Proof of Claim. Suppose $u(f(s)) - u(g(s)) \geq u(h(s)) - u(j(s))$ for every $s \in S$, and $h \succcurlyeq^* j$. For each $\lambda \in (0,1)$, let $\tilde{g}^\lambda \in \mathcal{F}$ be such that $\lambda u(j(s)) + (1-\lambda)u(\tilde{g}^\lambda(s)) = u(g(s))$ for every $s \in S$. Claim 2 guarantees that such a $\tilde{g}^\lambda$ exists. Define $g^\lambda := \lambda j + (1-\lambda)\tilde{g}^\lambda$ and $f^\lambda := \lambda h + (1-\lambda)\tilde{g}^\lambda$. By construction, $\lambda u(f(s)) + (1-\lambda)u(g(s)) \geq u(f^\lambda(s))$ and $u(g(s)) = u(g^\lambda(s))$ for every $s \in S$. By the second part of our independence axiom, we have that $f^\lambda \in c(\{f^\lambda, g^\lambda\}, g^\lambda)$. Now let $\hat{f}^\lambda$ be any act such that $u(\hat{f}^\lambda(s)) > u(f^\lambda(s))$ for every $s \in S$. Again, Claim 2 guarantees that such an act $\hat{f}^\lambda$ exists. Now fix $\alpha \in (0,1)$. By WARP and Dominance, it must be the case that $\alpha f^\lambda + (1-\alpha)\hat{f}^\lambda \in c(\{\alpha f^\lambda + (1-\alpha)\hat{f}^\lambda, f^\lambda, g^\lambda\}, g^\lambda)$. By (4) and WARP, this now implies that $g^\lambda \notin c(\{\alpha f^\lambda + (1-\alpha)\hat{f}^\lambda, g^\lambda\}, g^\lambda)$. Now Dominance and WARP imply that $g \notin c(\{\alpha f^\lambda + (1-\alpha)\hat{f}^\lambda, g\}, g)$. Since α above was chosen arbitrarily, Continuity now implies that $f^\lambda \in c(\{f^\lambda, g\}, g)$. Applying Dominance and WARP once more, we learn that $\lambda f + (1-\lambda)g \in c(\{\lambda f + (1-\lambda)g, g\}, g)$. Since this is valid for any $\lambda \in (0,1)$, one last application of Continuity now implies that $f \in c(\{f, g\}, g)$. □

Now note that saying that an alternative y reference-dominates another alternative x is equivalent to saying that there exists $f, g \in \mathcal{F}$ and $s \in S$ such that $f \succcurlyeq^* g$, $x\{s\}f \succcurlyeq y\{s\}g$, but it is not true that $x\{s\}f \succcurlyeq^* y\{s\}g$. We now show that there exists an alternative y that reference-dominates another alternative x if and only if $\succcurlyeq$ and $\succcurlyeq^*$ are different relations.

Claim 5. *There exists an alternative y that reference-dominates another alternative x if and only if $\succcurlyeq \neq \succcurlyeq^*$.*

Proof of Claim. It is clear that if $\succcurlyeq = \succcurlyeq^*$ then there exists no pair (x, y) with y reference-dominating x. Suppose now that for no $x, y \in X$ it is true that y reference-dominates x. By SQI, we know that, for any two acts f and g, $f \succcurlyeq^* g$ implies $f \succcurlyeq g$, so we only need to show that $f \succcurlyeq g$ implies $f \succcurlyeq^* g$. Fix any pair of acts f and g such that $f \succcurlyeq g$. Let $\{s_1, ..., s_n\}$ be any enumeration of the states in S where the states in which $u(f(s)) \geq u(g(s))$ appear first. Fix any pair of acts $\hat{f}$ and $\hat{g}$ such that $u(\hat{f}(s)) \geq u(\hat{g}(s))$ for all $s \in S$ and note that, since $\hat{g} \succcurlyeq^* \hat{g}$, Claim 4 implies that $\hat{f} \succcurlyeq^* \hat{g}$. For each $i \in \{1, ..., n\}$ let $f^i := f\{s_1, ..., s_i\}\hat{f}$ and $g^i := g\{s_1, ..., s_i\}\hat{g}$. By construction, for all $i \in \{1, ..., n\}$ we have $f^i \succcurlyeq g^i$. Now a simple inductive argument guarantees that $f^i \succcurlyeq^* g^i$ for all i and, in particular, $f = f^n \succcurlyeq^* g^n = g$. □

Using the claim above, WARP and SQI we can show that if for no pair (x, y) it is true that y reference-dominates x, then $c(A, g) = c(A, \diamond)$ for all $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$. In this case, if we define $\theta := 0$ and pick any $\gamma \in \mathbb{R}_+$ we get the desired representation. From now on, we assume that there exists some pair of alternatives x and y such that y reference-dominates x. Let $\gamma := \inf\{u(y) - u(x) : y \text{ reference-dominates } x\}$. We need the following three claims:

Claim 6. *For any $x, y \in X$, if $u(y) - u(x) > \gamma$, then, for any two acts f and g with $f \succcurlyeq g$ and any state $s^* \in S$, if it is not true that $f \succcurlyeq^* g$, then it is not true that $x\{s^*\}f \succcurlyeq^* y\{s^*\}g$.*

Proof of Claim. Suppose $u(y) - u(x) > \gamma$. By the definition of γ, there exists an alternative $\hat{y}$ that reference-dominates another alternative $\hat{x}$ such that $u(y) - u(x) > u(\hat{y}) - u(\hat{x})$. Let $f, g \in \mathcal{F}$ be such that $f \succcurlyeq g$, but it is not true that $f \succcurlyeq^* g$ and fix some $s^* \in S$. Suppose $x\{s^*\}f \succcurlyeq^* y\{s^*\}g$. Since $u(\hat{x}) - u(\hat{y}) > u(x) - u(y)$, Claim 4 implies that $\hat{x}\{s^*\}f \succcurlyeq^* \hat{y}\{s^*\}g$, which contradicts Reference-Dominance Coherence. We conclude that it is not true that $x\{s^*\}f \succcurlyeq^* y\{s^*\}g$. □

Claim 7. *$\gamma \geq 0$.*

Proof of Claim. To show that $\gamma \geq 0$, it is enough to show that for any alternative y that reference-dominates another alternative x we have $u(y) > u(x)$. For that, suppose that y reference-dominates x and $u(x) \geq u(y)$. Pick any pair of acts f and g such that $f \succcurlyeq g$, but it is not true that $f \succcurlyeq^* g$. Let $\{s_1, ..., s_n\}$ be any enumeration of the states in S where the states in which $u(g(s)) > u(f(s))$ appear first. By construction, $x\{s_1, ...s_i\}f \succcurlyeq y\{s_1, ..., s_i\}g$ for every $i \in \{1, ..., n\}$. However, an inductive argument based on Reference-Dominance Coherence implies that for no $i \in \{1, ..., n\}$ we have $x\{s_1, ...s_i\}f \succcurlyeq^* y\{s_1, ..., s_i\}g$. In particular, it is not true that $x \succcurlyeq^* y$, which contradicts Claim 4. We conclude that whenever y reference-dominates x we have $u(y) > u(x)$ and, consequently, $\gamma \geq 0$. □

Claim 8. *If an alternative y reference-dominates another alternative x, then $u(y) - u(x) > \gamma$.*

Proof of Claim. Suppose $u(y) - u(x) = \gamma$ and pick any $z \in X$ with $u(z) > u(x)$. Consider any pair of acts f and g, and state $s^* \in S$ such that $f \succcurlyeq^* g$ and $x\{s^*\}f \succcurlyeq y\{s^*\}g$. By the definition of γ, y does not reference-dominate $\lambda z + (1-\lambda)x$ for any $\lambda > 0$, which implies that $(\lambda z + (1-\lambda)x)\{s^*\}f \succcurlyeq^* y\{s^*\}g$ for every $\lambda > 0$. By Continuity, it must be the case that $x\{s^*\}f \succcurlyeq^* y\{s^*\}g$, which shows that y does not reference dominate x. $\square$

Now define an event $T \subseteq S$ to be blocking if there exists f, g such that $f \succcurlyeq g$, it is not true that $f \succcurlyeq^* g$ and $T = \{s \in S : u(g(s)) - u(f(s)) > \gamma\}$. We need the following claim:

Claim 9. *For any two acts f and g, and any blocking event T, if $\{s \in S : u(g(s)) - u(f(s)) > \gamma\} = T$, then it is not true that $f \succcurlyeq^* g$.*

Proof of Claim. Since T is blocking, there exists $\hat{f}, \hat{g} \in \mathcal{F}$ such that $\hat{f} \succcurlyeq \hat{g}$, it is not true that $\hat{f} \succcurlyeq^* \hat{g}$ and $T = \{s \in S : u(\hat{g}(s)) - u(\hat{f}(s)) > \gamma\}$. By Claim 6, we may assume, without loss of generality, that $u(\hat{g}(s)) - u(\hat{f}(s)) < u(g(s)) - u(f(s))$ for each $s \in T$. We first show that it cannot be true that $\hat{f}Tf \succcurlyeq^* \hat{g}Tg$. To see that, suppose this was true. Remember that, by SQI, this implies that $\hat{f}Tf \succcurlyeq \hat{g}Tg$. Enumerate the states in $S \setminus T$ in any way such that the states where $u(\hat{f}(s)) - u(\hat{g}(s)) \geq u(f(s)) - u(g(s))$ come first. Write $\{s_1, ..., s_n\}$ to represent this enumeration. Note that, by construction, for any $i = 1, ..., n$, we have $\hat{f}T \cup \{s_1, ..., s_i\}f \succcurlyeq \hat{g}T \cup \{s_1, ..., s_i\}g$. But then, there is $i \in \{1, ..., n\}$ such that $\hat{f}T \cup \{s_1, ..., s_{i-1}\}f \succcurlyeq^* \hat{g}T \cup \{s_1, ..., s_{i-1}\}g$, $\hat{f}T \cup \{s_1, ..., s_i\}f \succcurlyeq \hat{g}T \cup \{s_1, ..., s_i\}g$, but it is not true that $\hat{f}T \cup \{s_1, ..., s_i\}f \succcurlyeq^* \hat{g}T \cup \{s_1, ..., s_i\}g$.[19] This implies that $\hat{g}(s_i)$ reference-dominates $\hat{f}(s_i)$, which contradicts Claim 8. We conclude that it cannot be true that $\hat{f}Tf \succcurlyeq^* \hat{g}Tg$. Now note that, for each $s \in T$, $u(\hat{f}(s)) - u(\hat{g}(s)) > u(f(s)) - u(g(s))$. If it were true that $f \succcurlyeq^* g$, Claim 4 would imply that $\hat{f}Tf \succcurlyeq^* \hat{g}Tg$, which we know it is not true. We conclude it cannot be true that $f \succcurlyeq^* g$. $\square$

Now define the collection of events $\mathcal{T}$ by $\mathcal{T} := \{T \subseteq S : \exists f, g \in \mathcal{F}$ with $\{s \in S : u(g(s)) - u(f(s)) \leq \gamma\} = T$ and $f \succcurlyeq^* g\}$. By SQI and the definition of $\mathcal{T}$, for any two acts f and g, if $f \succcurlyeq^* g$, then $f \succcurlyeq g$ and $\{s \in S : u(g(s)) - u(f(s)) \leq \gamma\} \in \mathcal{T}$. We note that, by Claim 9, the converse is also true. That is, if $f \succcurlyeq g$ and $\{s \in S : u(g(s)) - u(f(s)) \leq \gamma\} \in \mathcal{T}$, then $f \succcurlyeq^* g$. Now, Claim 3 implies that, for every $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A, g) = \underset{f \in \mathcal{D}(A,g,\pi,\gamma,\mathcal{T})}{\arg\max} E_\pi [u(f)],$$

[19]When $i = 1$, we are using the convention that the acts $\hat{f}T \cup \{s_1, ..., s_{i-1}\}f$ and $\hat{g}T \cup \{s_1, ..., s_{i-1}\}g$ are the acts $\hat{f}Tf$ and $\hat{g}Tg$, respectively.

where, for each $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$\mathcal{D}(A, g, \pi, \gamma, \mathcal{T}) := \{f \in A : \{s : u(f(s)) \geq u(g(s)) - \gamma\} \in \mathcal{T}\}.$$

To complete the proof of the theorem, we need the following claim:

Claim 10. *There exists $\theta \in [0, 1]$ such that, for any $T \subseteq S$, $T \in \mathcal{T}$ if and only if $\pi(T) \geq \theta$.*

Proof of Claim. Fix some event $T \in \mathcal{T}$ and some event $\hat{T} \notin \mathcal{T}$. Now pick alternatives $x, y, z \in X$ such that $u(x) > u(y) > u(z) + \gamma$ and $\pi(T)u(x) + (1 - \pi(T))u(z) > u(y)$. Let $f := xTz$ and $g := x\hat{T}z$. By the claims above, we must have $\{f\} = c(\{f, g, y\}, y)$. But, by Binary Consistency, this implies that $\{f\} = c(\{f, g, y\}, \diamond)$, which implies that $\pi(T) > \pi(\hat{T})$. Notice that T and $\hat{T}$ were entirely arbitrary in the analysis above, so if we define $\theta := \min\{\pi(T) : T \in \mathcal{T}\}$ we have the desired characterization of $\mathcal{T}$. □

Finally, observe that the claim above implies that there exists $\theta \in [0, 1]$ such that, for any $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, we have that $\mathcal{D}(A, g, \pi, \gamma, \mathcal{T}) = \mathcal{D}(A, g, \pi, \theta, \gamma)$, which gives us the desired representation. ■

A.2. **Proof of Proposition 1.** By the uniqueness of subjective expected utility representations we know that $\pi = \hat{\pi}$ and that there exist $\alpha \in \mathbb{R}_{++}$ and $\beta \in \mathbb{R}$ such that $\hat{u} = \alpha u + \beta$. We can assume, without loss of generality, that in fact $u = \hat{u}$.[20] Since $c(A, \diamond) \neq c(A, g)$ for some $c(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, there must exist a state $s \in S$ such that $0 < \pi(s) < \theta$. This implies that there exists an event $T \subseteq S$ and a state $s^* \in S$ such that $0 < \pi(T) < \theta$, but $\pi(T \cup \{s^*\}) \geq \theta$. Let $x, y, z \in X$ be such that $u(y) - u(x) > \gamma$, $u(y) - u(z) \leq \gamma$ and $u(z)\pi(T) + u(x)(1 - \pi(T)) \geq u(y)$. Since (u, π, θ, γ) represents c, we must have $zTx \notin c(\{zTx, y\}, y)$, but $z(T \cup \{s^*\})x \in c(\{z(T \cup \{s^*\})x, y\}, y)$. Since $(u, \hat{\pi}, \hat{\theta}, \hat{\gamma})$ also represents c, this can happen only if $u(y) - u(x) > \hat{\gamma}$. Since x and y were arbitrarily chosen, this implies that $\hat{\gamma} \leq \gamma$. A symmetrix argument shows that $\gamma \leq \hat{\gamma}$. It remains to show that $\mathcal{T} = \hat{\mathcal{T}}$. Fix $T \in \mathcal{T}$. Pick alternatives $x, y, z \in X$ such that $\pi(T)u(z) + (1 - \pi(T))u(x) \geq u(y)$ and $u(z) > u(y) > u(x) + \gamma$. Since $T \in \mathcal{T}$, this implies that $zTx \in c(\{zTx, y\}, y)$. But this can be true only if $T \in \hat{\mathcal{T}}$. We conclude that $\mathcal{T} \subseteq \hat{\mathcal{T}}$. A symmetric argument shows that $\hat{\mathcal{T}} \subseteq \mathcal{T}$. ■

[20]It is clear that $(\hat{u}, \pi, \theta, \alpha\gamma)$ also represents c, so we can use this representation to finish the proof.

A.3. **Proof of Proposition 2.** It is clear that (2) $\Rightarrow$ (1). We will now show that (1) $\Rightarrow$ (2). For that, suppose that c_1 reveals more bias towards the status quo than c_2 and pick a representation $(u, \pi, \theta_1, \gamma_1)$ of c_1 with $\gamma_1 > 0$ and a representation $(u_2, \pi_2, \theta_2, \gamma_2)$ of c_2. Since c_1 and c_2 are non-trivial, neither u nor u_2 are constant and, consequently, if (u, π) and (u_2, π_2) represent different Anscombe-Aumann preferences over $\mathcal{F}$, then there exist acts f and g such that

$$E_\pi [u(f)] > E_\pi [u(g)] ,$$

but

$$E_{\pi_2} [u_2(g)] > E_{\pi_2} [u_2(f)] .$$

Since $\gamma_1 > 0$, it is clear from the representation of c_1 that, for λ sufficiently small, $\lambda f + (1 - \lambda)g \in c_1(\{\lambda f + (1 - \lambda)g, g\}, g)$. But from the representation of c_2 we have $\{g\} = c_2(\{\lambda f + (1 - \lambda)g, g\}, g)$, which contradicts the fact that c_1 reveals more bias towards the status quo than c_2. We conclude that (u, π) and (u_2, π_2) represent the same Anscombe-Aumann preferences over $\mathcal{F}$ and, therefore, $\pi_2 = \pi$ and there exist $\alpha \in \mathbb{R}_{++}$ and $\beta \in \mathbb{R}$ such that $u_2 = \alpha u + \beta$. We can now assume, without loss of generality, that $u_2 = u$ and $\pi_2 = \pi$. If $c_2(A, g) = c_2(A, \diamond)$ for all $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, then any $\gamma \geq 0$ and any $\theta \leq \theta_2$ can be used to represent c_2, in particular, $\gamma = \max(\gamma_1, \gamma_2)$ and $\theta = \min(\theta_1, \theta_2)$. Suppose, then, that $c_2(A, g) \neq c_2(A, \diamond)$ for some $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$. This implies that there exists an event $T \in 2^S \setminus \{\emptyset\}$ such that $0 < \pi(T) < \theta_2$. Suppose also that $\gamma_1 > \gamma_2$. Pick alternatives $x, y, z \in X$ such that $u(x) + \gamma_1 > u(y) > u(x) + \gamma_2$ and $\pi(T)u(z) + (1 - \pi(T))u(x) \geq u(y)$. Notice that this implies that $zTx \in c_1(\{zTx, y\}, y)$, but $zTx \notin c_2(\{zTx, y\}, y)$, which contradicts the fact that c_1 reveals more bias towards the status quo than c_2. We conclude that $\gamma_2 \geq \gamma_1$. Now suppose that $\theta_2 > \theta_1$. Suppose also that there exists an event $T \in 2^S \setminus \{\emptyset\}$ such that $\theta_2 > \pi(T) \geq \theta_1$. Pick alternatives $x, y, z \in X$ such that $u(y) > u(x) + \gamma_2$ and $\pi(T)u(z) + (1 - \pi(T))u(x) \geq u(y)$. By the representation of c_1, we have $zTx \in c_1(\{zTx, y\}, y)$, while, by the representation of c_2, we have $zTx \notin c_2(\{zTx, y\}, y)$. This again contradicts the fact that c_1 reveals more bias towards the status quo than c_2. We conclude that $\mathcal{T}_{\theta_1} = \mathcal{T}_{\theta_2}$ and, consequently, $(u, \pi, \min(\theta_1, \theta_2), \max(\gamma_1, \gamma_2))$ represents c_2. ■

A.4. **Proof of Proposition 3.** (a) and (b) are straightforward and the proof is omitted. To see that (c) holds, if u, π, γ and $\theta = 0$ represent c, then c simply maximizes expected utility and $S_c(f) = B_c(f)$ for all $f \in \mathcal{F}$. This shows that (1) implies (3). Also, it is clear that if, for all $s \in S$, $\pi(s) > 0$ implies $\pi(s) \geq \theta$, then u,π, γ and $\theta = 0$ also represent c. That is, (3) implies to (2). So we only have to show that (2) implies (1). Suppose that there exists $s^* \in S$ such that $\theta > \pi(s^*) > 0$. Pick $x, y, z \in X$ such that $u(x) > u(y) > u(z) + \gamma$ and $\pi(s^*)u(x) + (1 - \pi(s^*))u(z) > u(y)$. Consider the act f such that $f(s^*) = x$ and $f(s) = z$ for all $s \neq s^*$. Notice that $E_\pi[u(f)] > u(y) > \max\{v \in \mathbb{R} : \pi\{s : u(f(s)) + \gamma \geq v\} \geq \theta\}$. Since $S_c(f) \geq E_\pi[u(f)]$ and $B_c(f) \leq \max\{v \in \mathbb{R} : \pi\{s : u(f(s)) + \gamma \geq v\} \geq \theta\}$, we conclude that $S_c(f) > B_c(f)$. ■

A.5. **Proof of Proposition 4.** Recall that the investor's problem can be written as

$$\max_{\beta_f, \beta_h} \beta_f E[f] + \beta_h E[h]$$

s.t.

$$\beta_h, \beta_f \geq 0$$

$$\beta_f V_f + \beta_h V_h = W$$

$$P\left\{\beta_f f + \beta_h h - \frac{W}{Vh} h \geq -\gamma\right\} \geq \theta.$$

We can write the third restriction as

$$P\left\{\beta_f f - \frac{(W - \beta_h V_h)h}{V_h} < -\gamma\right\} \leq 1 - \theta,$$

which, using the second restriction, can be written as

$$P\left\{\frac{f}{V_f} - \frac{h}{V_h} < \frac{-\gamma}{\beta_f V_f}\right\} \leq 1 - \theta.$$

Now the restriction above can be written as

$$\frac{-\gamma}{\beta_f V_f} \leq q_{1-\theta}\left(\frac{f}{V_f} - \frac{h}{V_h}\right).$$

Because of Assumption 1, we know that $q_{1-\theta}\left(\frac{f}{V_f} - \frac{h}{V_h}\right) < \frac{-\gamma}{W} < 0$ and, therefore, the expression above becomes

$$\beta_f \leq \frac{-\gamma}{V_f q_{1-\theta}\left(\frac{f}{V_f} - \frac{h}{V_h}\right)}.$$

Since $\frac{E[f]}{V_f} > \frac{E[h]}{V_h}$, it is clear that the restriction above will be binding and, therefore, the investor will buy $\beta_f = \frac{-\gamma}{V_f q_{1-\theta}\left(\frac{f}{V_f} - \frac{h}{V_h}\right)}$ of the foreign asset. ■

A.6. **Proof of Proposition 5.** Recall that in this case the investor's problem can be written as

$$\max_{\beta_h,\{\beta_j\}_{j=1}^J} \sum_{j=1}^{J} \beta_j E\left[f_j\right] + \beta_h E[h]$$

s.t.

$$\beta_h \geq 0, \beta_j \geq 0 \text{ for } j = 1, ..., J,$$

$$\sum_{j=1}^{J} \beta_j V_j + \beta_h V_h = W,$$

$$P\left\{\sum_{j=1}^{J} \beta_j f_j + \beta_h h - \frac{W}{V_h} h \geq -\gamma\right\} \geq \theta.$$

We note that we can write the final value of the investor's portfolio as

$$\frac{\left(\sum_{j=1}^J \beta_j f_j\right)\left(\sum_{j=1}^J \beta_j V_j\right)}{\sum_{j=1}^J \beta_j V_j} + \beta_h V_h \frac{h}{V_h} = \left(\sum_{j=1}^{J} \beta_j V_j\right) R + \beta_h V_h R_h.$$

Using the budget constraint the expression above can be written as

$$\left(W - \beta_h V_h\right) R + \beta_h V_h R_h.$$

Since R depends only on the relative weights of each asset in the international part of the investor's portfolio, we can write the investor's problem as

$$\max_{\beta_h,\{b_j\}_{j=1}^J} \left(W - \beta_h V_h\right)\left(E[R] - E[R_h]\right) + W E[R_h]$$

s.t.

$$\beta_h \geq 0, b_j \geq 0 \text{ for } j = 1, ..., J, \ \sum_{j=1}^{J} b_j = 1,$$

$$P\left\{R - R_h \geq \frac{-\gamma}{W - \beta_h V_h}\right\} \geq \theta.$$

The last restriction can be written as

$$P\left\{R - R_h < \frac{-\gamma}{W - \beta_h V_h}\right\} \leq 1 - \theta.$$

As we did before, define $q_{1-\theta}\left(R - R_h\right) := \max\left\{q : P\left\{R - R_h < q\right\} \leq 1 - \theta\right\}$. The restriction above is equivalent to

$$\frac{-\gamma}{W - \beta_h V_h} \leq q_{1-\theta}\left(R - R_h\right).$$

Given Assumption 2, the optimum portfolio will satisfy $E[R] > E[R_h]$ and will include a positive amount of the home asset. Therefore, if the restriction above was not binding the investor could invest a little less on the home asset, still satisfying the restriction, and increase her expected return. This shows that the best portfolio must satisfy

$$\frac{-\gamma}{W - \beta_h V_h} = q_{1-\theta}\left(R - R_h\right). \tag{5}$$

Using this condition, the investor's problem becomes

$$\max_{\{b_j\}_{j=1}^J} \frac{-\gamma}{q_{1-\theta}\left(R - R_h\right)}(E[R] - E[R_h]) + WE[R_h]$$

s.t.

$$b_j \geq 0 \text{ for } j = 1, ..., J, \ \sum_{j=1}^{J} b_j = 1.$$

This shows that finding a solution to the original problem is the same thing as finding a solution to the problem above and satisfying (5), which is equivalent to condition (ii) in the statement of Proposition 5. ■

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