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Experimentation in Organizations

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Abstract

We consider a moral hazard problem in which a principal provides incentives to a team of agents to work on a risky project. The project consists of two milestones of unknown feasibility. While working unsuccessfully, the agents’ private beliefs regarding the feasibility of the project decline. This learning requires the principal to provide rents to prevent the agents from procrastinating and free-riding on others’ discoveries. To reduce these rents the principal stops the project inefficiently early and gives identical agents asymmetric experimentation assignments. The principal prefers to reward agents with better contract terms or task assignments rather than monetary bonuses.

Keywords: principal-agent, moral hazard, experimentation, exponential bandit, contests.

JEL Codes: D82, D83, D86.

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1 Introduction

Innovation within firms typically involves groups of workers who coordinate their efforts and build on each others’ achievements to complete a common goal. Many potentially lucrative projects require a large amount of work, for which one individual’s labor will not suffice. Innovative projects are often uncertain, with many possible points of failure or steps that may fail to come through. As workers participate on risky but potentially lucrative projects, they may learn privately from their own experience and from their coworkers’ about their projects’ quality and feasibility. These sources of dynamic private information complicate the provision of incentives by a principal or a manager.

We develop a model of experimentation in organizations in which a manager (principal) contracts with a group of workers (agents) to complete a project. The project consists of two tasks of unknown feasibility, each of which has to be completed for the project to yield a final payoff. We model this setting as a sequence of experiments. The agents experiment simultaneously, and each agent has private information about his effort provision. Because efforts are privately observed, as the agents work they privately learn about the feasibility of each stage. When one worker completes a task, other workers may build on it in order to start working on the next stage of the project. The principal chooses a history-contingent payoff scheme to incentivize agents to exert effort at each time subject to providing in expectation a utility of at least each agent’s outside option and satisfying limited liability. The outside option represents the state of the economy when the agent is hired. We characterize the profit-maximizing contract under these circumstances.

The literature on contracts for experimentation focuses mainly on principal-agent relationships with a single agent in which all uncertainty is resolved after a single success. However, projects typically involve many milestones that need to be reached and have many possible points of failure. Workers in an organization interact through multiple stages until a project is abandoned or completed. Workers’ beliefs in the feasibility of the project will increase after they achieve milestones and decrease when time passes without progress. Our model allows us to ask how workers’ payments and terms of employment vary over time and to examine the effects on incentives and optimal contracts of learning from coworkers.

For example, consider a group of software developers working on the development of a new computer game. The workers will need to design the storyline and graphics, create a prototype, program the different stages of the game, test the game with different types of users and revolve issues and bugs. Each of these steps is uncertain but is crucial for the success of the new product. Workers’ beliefs in the feasibility of the project will increase after they achieve milestones and will

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1 See, for example, Bergemann and Hege (2005), Bergemann and Hege (1998), Hörner and Samuelson (2013) and Halac et al. (2016b).
decrease as time passes without progress. Moreover, if one worker develops the right concept or discovers how to overcome a hurdle, other workers will learn from it.

We show that, as long as any uncertainty remains about the feasibility of the project, agents must receive rents in all stages of the project in which they exert effort. These rents have two sources. The first source of rents is required to prevent agents from shifting the timing of their efforts. If the principal intends to implement a given effort schedule while providing no rents to the agents, he will have to fine tune the payments to each agent’s private information about the project. As the agents can control the rate at which they learn by shifting effort across time, they can shift their effort so that the principal’s payment is no longer fine-tuned and in fact gives the agents a rent. We refer to these rents as “procrastination rents.” An essential feature of the model is that the more effort the agents exert, the more they learn about the quality of the project. Therefore, procrastination rents are larger when agents are granted more leeway to experiment.

In addition, because agents receive a rent in every uncertain stage, early successes of their coworkers lead to stages with positive rents for all agents that the principal continues to employ. This feature makes experimentation in early stages more costly. The principal has the option not to employ unsuccessful agents after their coworker completes a task, but doing so would slow the project. She optimally chooses to employ them but at worse terms. As a result, agents must receive additional rents to prevent them from shading their efforts in expectation that some other agent completes a task. This “public-good rent” appears because of the public good nature of intermediate successes. It arises endogenously from the fact that the principal chooses to continue to employ an agent in future tasks after a coworker achieves as success. Both types of rents are driven by the ability of each agent to control his learning about the feasibility of the project in an uncertain environment. In contrast, in a project in which both tasks were known to be feasible ex-ante, the principal is able to induce any effort function without providing either type of rents.

The optimal contract, characterized by a differential equation, minimizes rents given the efforts it induces. The principal faces a trade-off between providing rents and implementing efficient experimentation. In order to maximize profits, the principal implements inefficiently low levels of experimentation. In a project with one or two tasks, experimentation is stopped inefficiently early to reduce the information rents needed to prevent agents from shifting effort over time. The optimal two-task contract is not the repetition of the one-task contract. In a project with two tasks the experimentation assignments of the agents who do not achieve breakthroughs are distorted down, so as to reduce the public-good rents that increase the costs of providing incentives in the first task. Under some conditions, the principal chooses to distort the first task experimentation unevenly across players, offering asymmetric contracts to symmetric agents.

The optimal contract has three interesting features. First, the principal prefers to compensate an agent with a higher outside option or reward an agent for a success in an early stage with
experimentation assignments that provide more procrastination rents, instead of granting a one-
time monetary transfer such as a signing or performance bonus. These transfers are granted only
when it would be inefficient to let the agent experiment further. Intuitively, the principal faces
the choice of rewarding an agent with bonus and/or with an assignment that involves a lengthier
experimentation period. She chooses the latter because an assignment that costs as much as a
transfer generates a surplus, arising from the agent’s experimentation, which has the potential of
producing a breakthrough. Agents that face better market conditions at the time of hire, that is,
agents with a higher outside options, see their allocations in all tasks, after all histories, undistorted
from the second best.

Second, agents’ contracts are sensitive to their performance in the early stages. Agents who
succeed early are rewarded with reduced competition, more opportunities to succeed, and higher
bonuses conditional on success later in the project. Agents who fail in early stages are assigned less
experimentation or are allocated to less valuable, low risk projects in later stages. Thus, symmetric
agents may end up with substantially different task allocations and earnings, even if nothing is
learned about their abilities.

Third, contracts may be ex-ante asymmetric even when the agents are identical. In a model
with two agents, an agent may be barred from participating in the project in the initial step or may
be allocated a smaller experimentation assignment which provides less rents. This result is due to
the public-good rents. When one agent’s experimentation assignment is reduced the other agent
has less incentives to free-ride on the agent’s work. Thus, when there is more than one uncertain
stage the principal may let agents sit idle instead of allowing them to participate in the project.
Contracts are more likely to be asymmetric when the first task is relatively safe and the value of
achieving the first breakthrough is not enough to justify a lengthy experimentation.

The expected payoff of the agent in the early task has an intuitive form. It can be decomposed
as the bonus wage in the one task project plus the payoff an agent would receive if he were to shirk
during the first task. Thus, the relative importance of shifting the timing of effort and public-good
for incentives determines the shape of the optimal contract. When procrastination is more costly to
the principal, the expected payoff of the agents tends to increase with the total effort exerted before
the first discovery. When public-good rents are more costly, the expected payoff of the agent tends
to decrease.

The incentive to delay effort is reduced as the number of agents involved in the project in-
creases. When more agents are present, an individual agent faces more competition. He is less
tempted to procrastinate, as other agents are more likely to achieve milestones while he shirks. In
contrast, the free-riding incentive increases with the number of agents in the early stages of the
project and decreases with the number of agents in the later stages of the project. Thus, it is always
optimal to add more agents in the last stage of the project but there may be no additional gain from
allowing these agents to participate in early stages.

An important question in the design of the optimal contract regards the optimal disclosure policy of agents’ successes. For example, would the principal prefer that agents work in isolation and sometimes experiment on a task that has already been completed or would she prefer to disclose publicly that an agent succeeded and allow agents to proceed to the next task as soon as a breakthrough is achieved? The former may be beneficial because it allows the principal to offer lower bonuses for breakthroughs: as agents do not learn about their coworker’s successes they are more optimistic about their ability to obtain them. We show that the optimal disclosure policy involves immediate disclosure to all agents and, thus, exhibits no duplication of effort.\(^2\) Our results have implications for the design of optimal contests for experimentation. If the principal does not have a binding budget constraint, the winner-takes-all is the optimal contest.

Our optimal contract has two features that are commonly observed in real-world contracts. First, firms often use job assignments or promotions to reward workers instead of only bonuses.\(^3\) If in a firm promotion involves a task allocation that must confer more information rents due to learning, our model shows that it may be optimal to reward an agent via promotions rather than bonuses. A promotion could be an assignment that is undistorted relative to the second best or an alternative task that provides more rents. Second, economic conditions at the time of hire have persistent effects on workers’ careers.\(^4\) In the model, when workers have a higher outside option, their allocations provide more information rents in all tasks and after all histories. Furthermore, they receive higher rewards for similar successes. Hence, their employment terms are persistently better during their tenure in the firm.

\(^2\)This result is in contrast with Halac et al. (2016a). In their paper, because the contest designer has a fixed budget to allocate as a prize, it is sometimes optimal to share the prize among all the agents who succeed and allow for duplication of effort.

\(^3\)Baker et al. (1988) pose the question of why promotions are so widely used to provide incentives in firms, given the common wisdom that bonuses should be preferable. An alternative explanation is given in Fairburn and Malcomson (1994) who show that promotions allow the manager to implement higher effort when it is possible for workers to bribe the manager. Prendergast (1993) models promotions as a way to provide incentives to make unobservable investments in specific human capital. See Gibbons and Waldman (1999) for an excellent survey relating theory and observed contracts.

\(^4\)Beaudry and DiNardo (1991) and Baker et al. (1994) find the presence of cohort effects in different contexts. The latter paper studies the records of one firm and finds that there was variability in the entrance wages of different cohorts and that these differences affected wages and raises persistently for several years. They also find that in a poor economy workers are assigned to lower level jobs within the firm. More recently, Kahn (2010) demonstrates persistent effects of poor labor conditions at the time entry into the labor force. Gibbons and Waldman (2006) presents a model in which task-specific human capital can account for cohort effects. In our model, we interpret labor conditions as the level of the outside option which has persistent effects because the principal prefers to compensate an agent who has a higher outside option with an allocation that provides more information rents. Workers who are hired in a bad economy are assigned jobs that provide less information rents. Baker et al. (1994) propose as a possible explanation of cohort effects that workers may be risk averse and therefore, a higher wage must be “smoothed” over time. In our model the smoothing motive is absent because the agents are risk neutral.
**Related Literature.** This work adds to the literature of experimentation, (see for instance Bolton and Harris (1999) Keller et al. (2005) and Klein and Rady (2011)), the literature on contests, and the literature of incentives for teams of agents under moral hazard.

Bonatti and Hörner (2011) analyze a game in which a group of players who have private information about their efforts collaborate to obtain a success in a risky project with one stage of uncertainty. The players receive the same payoff when they succeed. They find that the equilibria of the game have inefficient delays in provision of effort. Bonatti and Hörner (2009) ask the closely related question of what is the contract a principal would optimally offer the agents to complete their project if she cannot observe the individual successes. The main difference is that we assume that the principal can observe each player’s breakthroughs and can offer asymmetric contracts and there are two stages of uncertainty. The papers mentioned feature free-riding in experimentation. That is to say, in equilibrium the players shade their effort to learn from other agents’ breakthroughs. However, we consider a principal-agents setting. By constructing an optimal contract we characterize the minimum rents agents must receive to prevent free-riding and show how these affect the terms of employment and its dynamics for symmetric players. The characterization of these rents is new to the literature of experimentation.

Bergemann and Hege (1998), Bergemann and Hege (2005), and Hörner and Samuelson (2013) consider principal one-agent experimentation settings in which the principal must provide funds to an agent who may appropriate them. Thus, the contract must satisfy a “no diversion constraint”. In their setting there is one stage of uncertainty only. Green and Taylor (2015) and Hu (2014) consider a two-stage project with a “no diversion” constraint and no uncertainty about the quality of the arm. We depart from this literature by considering a multiple agent model with two stages of uncertainty in which the agents do not receive a flow of funding. Our limited liability constraint is weaker than “no diversion” and is intended to capture features of a firm that employs workers. Because our incentive constraints are weaker we find that under no uncertainty the principal can trivially implement efficient experimentation and extract all surplus for any number of tasks. Shan (2017) considers a model in which a principal must provide incentives for multi-stage team research involving risk-averse agents under varying effort complementarity assumptions. The main difference with our work is that in his setting there is no uncertainty about the feasibility of each stage and, therefore, the issue of procrastination rents does not arise.

Our paper also relates to the literature on contests. In complementary work, Halac, Kartik, and Liu (2016a) ask how to design a contest for experimentation for a group of symmetric agents. In their paper the principal maximizes the amount of experimentation subject to a budget constraint which bounds the maximum prize after each history. They find that it is sometimes optimal to not disclose breakthroughs to other participants.⁵ In our paper, in contrast, we solve for the

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⁵Note that the problem in Halac, Kartik, and Liu (2016a) is not the dual of the problem we consider. Our problem is
revenue maximizing contest without a budget constraint. To do so we characterize the expected-cost-minimizing contract for a given level of experimentation. We find that in the single milestone project, the cost minimizing contest for a given amount of experimentation discloses breakthroughs immediately and features no duplicated effort.

Bimpikis et al. (2014) analyze the design of an optimal contest with two uncertain stages. There are two main differences that distinguish our papers. First, they assume that each agent has to complete all milestones while we allow for policies that let contestants build on the milestones attained by others. Second, they restrict attention to rewards of constant size that do not depend on the time at which the intermediate milestones are reached while we allow for a more general space of contracts. We find that the optimal contract conditions on these additional variables.

The paper is also related to the literature on efficiency wages (Shapiro and Stiglitz (1984); Acemoglu and F. Newman (2002)). Efficiency wages arise when the principal has an imperfect monitoring technology and punishments for shirking are subject to limited liability. The limited liability constraint together with the incentive compatibility implies that the agent receives a rent. These setups yield a relationship between the ability to monitor and the rents allocated to the agent. In our model, the principal does not have the ability to monitor effort. The agents receive rents to prevent them from shifting effort over time, as they learn about the quality of their task.

The paper also contributes to a literature on contracting with a single agent and unobserved states and private effort. He et al. (2014) consider a principal-agent problem with moral hazard in which there is uncertainty about the project’s profitability. Because the principal does not observe the agent’s effort, the agent can manipulate the principal’s beliefs about the project’s profitability, leading to informational rents. Prat and Jovanovic (2014) and Bhaskar (2014) consider other moral hazard settings in which an agent can manipulate a principal’s beliefs by choice of effort, leading to informational rents. Halac et al. (2016b) characterize optimal contracts between a single agent and a principal in discrete time without limited liability. In their model the agent privately observes his own effort and type. Adverse selection in conjunction with moral hazard gives rise to inefficiencies and information rents to the agents.

Manso (2011) and Ederer (2013) consider a setting in which agents can privately choose between a safe and a risky action. The principal does not know which action produced a success. Other papers consider incentives in teams. Campbell et al. (2014) model a game with multiple agents and multiple breakthroughs which are privately observed, but without uncertainty about the quality of the project. Georgiadis (2015) presents a model of project and team dynamics in which the commonly observed state of the project evolves according to a controlled stochastic process.
driven by a Brownian motion. Georgiadis et al. (2014) consider the problem of a principal with limited commitment power managing a team of workers. Wagner (2016) considers an experimentation setting with multiple agents who are each assigned to different imperfectly correlated one-stage projects.

2 Model

There are $n \in \{1, 2\}$ symmetric agent(s) that participate in a project that is owned by a principal. One can think of the agents as workers that are employed by a firm (the principal) seeking to create a new product. The project consists of 2 stages or tasks each of which has to be completed in sequential order for the project to succeed. However, it is not known ex-ante whether each task can be completed. A task may be “good” or “bad” (or else “feasible” and “impossible”). The probability that task $j$ is good is $\bar{p}^j \in (0, 1)$ which is commonly known by all participants.

Time is continuous in $[0, \infty)$. At each time $t$ agents exert privately observed and costly effort in the current task. If task $j$ is being performed at time $t$, each agent $i$ exerts effort $a^j_{i,t} \in [0, \bar{a}]$ at time $t$ at cost $\kappa^j a^j_{i,t}$, where $\kappa^j > 0$.

Agents can start work on task 2 only after task 1 is completed successfully by some agent. If task $j$ is good and agent $i$ exerts effort $a^j_{i,t}$ at time $t$, he completes the task with instantaneous probability $a^j_{i,t}$. If task $j$ is bad it can never be completed, no matter how much effort the players put in.

We refer to the completion of task $j$ as achieving a breakthrough on task $j$. When a breakthrough is achieved in task $j$, the principal receives an instantaneous payoff $\pi^j$ (not necessarily positive). A breakthrough in task 2 has a value of $\pi^2 > 0$. As long as no breakthrough has occurred the principal does not reap any benefits from the project. All players discount the future at a common rate $r \geq 0$. We assume that the game ends after the second breakthrough.

The principal observes a breakthrough when it is achieved and the identity of the agent who achieved it. In the bulk of the paper we assume that the agents observe breakthroughs perfectly as well. We show in sections (3.2) and (4.3) that, under some conditions, the principal cannot improve on the optimal contract by designing the information disclosure of the other agent’s breakthroughs.

The set of public histories at time $t$ is denoted $\mathcal{H}^t$. A public history specifies which tasks have produced breakthroughs, the times at which breakthroughs were attained, and who attained each

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6In particular, agent $i \in \{1, 2\}$ is able to start the second task after $-i$ achieves a breakthrough.

7$\pi^1$ may be negative when beginning the second stage of a project requires the principal to purchase equipment or capital of some form.

8An alternative interpretation is that after the second breakthrough there is any number of tasks with known feasibility. When a task is known to be good the principal can implement any effort function by paying a bonus to an agent when he achieves a breakthrough. This bonus exactly compensates each agent for the cost of effort in expectation and, therefore, provides no rents.
breakthrough. Formally a history $h^t \in \mathcal{H}^t$ contains a sequence of time and agent pairs $(\tau^j, k^j)$ for $j \leq 2$. $\tau^j \leq t$ is the time at which the $j$’th breakthrough was attained by agent $k^j$. We denote $\mathcal{H}$, the set of realized (terminal) histories of breakthroughs. A history $h \in \mathcal{H}$ with $J \in \{0, 1, 2\}$ breakthroughs contains a sequence of time-agent pairs $\{(\tau^j, k^j)\}_{j=1}^{J}$ corresponding to the breakthroughs that were attained throughout the game (where $h = \emptyset$ if $J = 0$). Let $\mathcal{H}^t$ denote the set of public histories at time $t$ in which some breakthrough is attained at time $t$.

At the beginning of the game the principal offers each agent a wage schedule that is contingent on the public history. The contracts offered are publicly observed, and the principal has the ability to fully commit to these contracts. As shown in Appendix A.1, it is without loss to restrict attention to contracts that compensate agents only at time zero and at histories in which a breakthrough is achieved. We refer to these contracts as bonus contracts. Formally, a bonus contract specifies for every history $h^t \in \mathcal{H}^t$ a transfer $w_{i,t}(h^t) \in \mathbb{R}$ to each agent $i$ at time $t$ and a transfer to each agent $i$ at time zero, $W_{i,0}$. Bonus contracts provide no transfers nor flow payoffs at histories $h^t \notin (\mathcal{H}^1 \cup \mathcal{H}^0)$.

Even though the agents are symmetric, we allow the principal to offer different contracts to different agents.

Let $\mathcal{H}_{i}^{j,t}$ be the private history of agent $i$ at time $t$ in task $j$, consisting of the public history and the effort exerted by agent $i$ up to time $t$. Agent $i$’s strategy is a measurable function $a_{i,t}^{j} : \mathcal{H}_{i}^{j,t} \rightarrow [0, \bar{a}]$ from private histories to pure actions. $a_{i,t}^{j}(h^t)$ is the instantaneous effort that agent $i$ exerts at time $t$ in task $j$, at private history $h^t \in \mathcal{H}_{i}^{j,t}$. If the agents are working in task $j$ the effort in task $j$ must be zero. That is, at history $h^t \in \mathcal{H}_{i}^{k,t}$, for $k \neq j$, $a_{i,t}^{j}(h^t) = 0$.

The payoffs of the players are as follows. Consider a history $h = \{(\tau^j, k^j)_{j=1}^{J}\} \in \mathcal{H}$. Under history $h$, $J$ tasks were completed at times $\{\tau^j\}_{j \leq J}$. Define $\tau^0 = 0$ and $\tau^{J+1} = \infty$ and let $w_{i,\tau^j}(h)$, for $j \leq J$, denote the realized bonuses received by agent $i$ at times $\{\tau^j\}_{j \leq J}$ under terminal history $h$. The payoff to the principal is:

$$\left( \sum_{j \leq J} (\pi^j - w_{i,\tau^j}(h)) e^{-r\tau^j} \right),$$

and agent $i$’s payoff from exerting effort $(a_{i,t}^{j})_{t \geq 0}$ for each task $j$ is:

$$\left( \sum_{j \leq J+1} \left( w_{i,\tau^j}(h) e^{-r\tau^j} - \int_{\tau^{j-1}}^{\tau^j} e^{-r\kappa} a_{i,s}^{j} ds \right) \right).$$

Each agent must receive expected utility of at least his outside option, $\bar{U}$, from the contractual relationship. We think of this outside option as an alternative employment opportunity available

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9Although we allow for bonus contracts to give negative transfers, optimal contracts will involve only positive transfers. Hence, the name “bonus contracts”. This definition is adapted from Halac et al. (2016b).
when the contract is signed. This alternative offer expires once an agent chooses his employer.\(^{10}\)

We interpret \(\bar{U}\) as a signal of the quality of the job market when a worker gets hired. When the economy is good there are more positions available which drives up the outside option \(\bar{U}\).

The wages offered by the principal define a game between the agents. We will look for Perfect Bayesian equilibria of this game. Namely, each agent \(i\) chooses \(a_{i,t}\) to maximize his expected payoff. Among the equilibria induced by a given contract, we will look for the one that maximizes the principal’s payoff subject to the constraint that the agents receive a payoff of at least \(\bar{U}\). The objective of the principal is to offer contracts to each agent so as to maximize her expected payoff. In the case in which \(r = 0\), we’ll focus on the equilibrium that corresponds to a limit of equilibria as \(r\) converges to zero.\(^{11}\)

As agents exert effort on task \(j\) with \(\bar{p}^j \in (0, 1)\) they become more pessimistic about the feasibility of the task. Conditional on strategies \((a_{1,t}^1, \ldots, a_{n,t}^n)\) on task \(j\) player \(i\)’s private belief that \(j\) is good at time \(t\), \(p_{i,t}^j\), evolves according to the differential equation

\[
\frac{dp_{i,t}^j}{dt} = p_{i,t}^j = -p_{i,t}^j (1 - p_{i,t}^j) a_{i,t}^j,
\]

where \(a_{i,t}^j = \sum_i a_{i,t}^j\) and \(p_{i,t+1}^j = \bar{p}^j\). On path the belief is common to all, as we restrict attention to pure actions, and it is denoted \(p_t\).

We assume throughout that the agents are subject to limited liability, that is, the principal cannot extract a negative transfer after any history. This assumption is reasonable for agents who are credit constrained, or cannot legally commit to the contract, as is typically the case in employment contracts.

**Definition 1.** A bonus contract satisfies *limited liability* if at every history \(h' \in \mathcal{H}^t\) the bonus \(w_{i,t}(h')\) received by each agent \(i\) at history \(h'\) is weakly positive.

It is without loss to restrict attention to bonus contracts under limited liability. The reason is that, for any contract that offers payoffs as a measurable function of histories, and any equilibrium under that contract, there exists a bonus contract, and an equilibrium under the bonus contract, that gives the same discounted payoff to all agents and the principal after every realized terminal history \(h \in \mathcal{H}\). If the original contract offers a weakly positive discounted sum of transfers after each terminal history, the associated bonus contract satisfies limited liability. These statements are

\(^{10}\)This assumption is reasonable, for example, in a setting in which there are hiring seasons. These hiring seasons are common in many industries and may arise when there are large fixed costs of evaluating candidates or training them for the specific job once hired. Alternatively, firms may balk at hiring a worker who wants to leave his current employment, absence new information, if those who leave are adversely selected. We abstract from these concerns.

\(^{11}\)This limit is well defined because, as we will see, at the optimal contract the equilibrium is unique and the equilibrium actions are continuous in \(r\).
shown formally in section A.1 of the appendix.

2.1 The first-best

We begin with the social planner’s problem that characterizes the efficient level of experimentation. The social planner maximizes the sum of payoffs of all players. Suppose task 1 was completed at time $\tau^1$. The social planner’s program at the beginning the second task is:

$$\tilde{\Pi}^2 = \max_{a^2_{i,t}} \sum_i \int_{\tau^1}^{\infty} (p^2 - \kappa^2)a^2_{i,t}e^{-\int_{\tau^1}^{s}(p^2 + r)ds} dt,$$

where the belief evolves according to $p^2 = -p^2(1-p^2)a^2$, $\dot{p}^2 = \ddot{p}^2$. The term $e^{-\int_{\tau^1}^{s}(p^2a^2ds}$ is the probability that no breakthrough has occurred yet and, therefore, $p^2a^2e^{-\int_{\tau^1}^{s}(p^2a^2ds}$, is the probability density that $i$ obtains the first breakthrough in task 2 at time $t$. The belief that the arm is good $p^2$ decreases over time as long as no breakthrough has occurred, and its time derivative is proportional to the aggregate effort exerted by all agents. $\tilde{\Pi}^2$ is the planner’s expected payoff from experimentation in the second task at time $\tau^1$.\(^{12}\)

The solution to the second task program starting from time $\tau^1$ is to set $a^2_{i,t} = \bar{a}$ whenever $p^2 > \kappa^2$ and $a^2_{i,t} = 0$, otherwise, which determines $\tilde{\Pi}^2$.

At time zero, the social planner solves

$$\tilde{\Pi}^1 = \max_{a^1_{i,t}} \sum_i \int_{0}^{\infty} (p^1 + \tilde{\Pi}^2 - \kappa^1)a^1_{i,t}e^{-\int_{0}^{s}(p^1a^1 + r)ds} dt,$$

where the belief evolves according to $p^1 = -p^1(1-p^1)a^1$, $\dot{p}^1 = \ddot{p}^1$.

The integrand is positive if and only if $p^1 + \tilde{\Pi}^2 > \kappa^1$. Therefore, the solution to the planner’s program is a threshold strategy for each agent: $a^1_{i,t} = \bar{a}$ when $p^1 + \tilde{\Pi}^2 > \kappa^1$ and $a^1_{i,t} = 0$ when $p^1 + \tilde{\Pi}^2 \leq \kappa^1$. Each agent exerts effort as long as the expected marginal gain from effort is above its marginal cost.

Thus, under the social planner’s solution the agents work at full speed in task $j$ up to time

$$\tilde{T}^j = \frac{-\ln \left( \frac{1-p^j}{\bar{p}^j} \right) + \ln \left( \frac{\kappa^j + \tilde{\Pi}^j+1}{\kappa^j} \right)}{n\bar{a}}, \quad (1)$$

where $\tilde{\Pi}^3 = 0$. $\tilde{T}^j > 0$ whenever $\bar{p}^j \left( \pi^j + \tilde{\Pi}^j+1 \right) > \kappa^j$. The total amount of work exerted conditional on no breakthrough is given by $-\ln \left( \frac{1-p^j}{\bar{p}^j} \right) + \ln \left( \frac{\pi^j + \tilde{\Pi}^j+1}{\kappa^j} \right)$. This amount does not depend on whether there are one or two agents nor on their maximum effort $\bar{a}$. As one would

\(^{12}\)A change of variables in the planner’s objective shows that $\tilde{\Pi}^2$ does not depend on $\tau^1$.\]
expect, the total amount of work is also decreasing in the cost of effort $\kappa^j$ and increasing in the initial belief $\bar{p}^j$.

2.2 Procrastination rents

Before characterizing the optimal contract, let us give an intuition for why under limited liability the agents must receive payments in excess of their cost of effort. These excess payments are, in effect, information rents. For the agents to receive no rents while exerting effort, each agent $i$ must receive a bonus $w_{i,t}^{NR}$ such that $p_t w_{i,t}^{NR} - \kappa = 0$, where $p_t$ is the belief that the task is good given the effort exerted up to time $t$. If agent $i$ receives less than $w_{i,t}^{NR}$, he would not exert effort at time $t$. We call $w_{i,t}^{NR}$ the no-rent contract.

No non-zero effort function $a_{i,t}$ is incentive compatible under $w_{i,t}^{NR}$ because agent $i$ can guarantee a strictly positive payoff by exerting less effort than what he is expected to exert in some time interval. During the time interval in which no effort is exerted the payoff is zero, which is the same as he gets in the no-rent contract at the allocated effort. After the interval the agent is more optimistic about obtaining a success than he would have been if he had exerted the allocated effort. Since the full-rent contract makes an agent that has behaved as expected just indifferent between exerting effort or not, it must give an agent who is more optimistic than expected a strictly positive payoff.\(^1\),\(^2\)

The agents receive rents because they would like to delay their effort, under the no-rent contract. For this reason we call these information rents procrastination rents.

In contrast, if there is no uncertainty about the feasibility of the task, i.e. $p_t = 1$ for all $t$, the principal can induce any level of effort by setting $w_{i,t}^{NR} = \kappa$ for each player $i$. Under $w_{i,t}^{NR}$, $i$ is indifferent between all effort levels at all times.

\(^{13}\)The belief $p_t$ at time $t$ is given by $p_t = \frac{\bar{p}^t e^{-\int a_t dt}}{\bar{p}^t e^{-\int a_t dt} + (1-\bar{p}^t)}$. Thus, $p_t$ depends on the unobservable effort exerted by all agents up to time $t$.

\(^{14}\)More formally, consider the dynamic programming problem of the agent. Let $V_{i,t}$ denote the expected payoff of agent $i$ at time $t$. $V_{i,t}$ must satisfy

$$V_{i,t} = (p_t w_{i,t}^{NR} - \kappa) a_{i,t} dt + (1 - (r + p_t (a_{i,t} + a_{-i,t}))) dt V_{i,t+dt} + o(dt).$$

Under $w_{i,t}^{NR}$, agent $i$ gets zero payoff at every time when exerting effort according to $a_{i,t}$, and therefore $V_{i,t+dt} = 0$. Suppose $a_{i,t} > 0$ in an interval around $t$. If $i$ were to stop working for an instant at $t$ his private belief about the state of world would be strictly above $\kappa/w_{i,t}$ for every $\tau > t$ and later effort would give him a strictly positive payoff, obtaining $V_{i,t+dt} > 0$. At time $t$ agent $i$ obtains zero payoff by setting $a_{i,t} = 0$ which is the same he obtains by exerting effort $a_{i,t} > 0$. Thus, under $w_{i,t}^{NR}$, agent $i$ has incentives to shift effort to the future knowing that he will be more optimistic about the state of the world at that time.

\(^{15}\)This effect is also present in the models found in Bergemann and Hege (1998); Bonatti and Hörner (2011); Hörner and Samuelson (2013) and Bhaskar (2014).
3 Benchmark: project with a single task

In this section we characterize the optimal contract in the benchmark one-task project case. As there is only one task we omit the task superscripts.\textsuperscript{16} Let $w_{i,t}$ denote the transfer agent $i$ receives when he achieves a breakthrough at time $t$. Paying an agent for another agent’s success reduces the incentives to exert effort, and negative transfers are not permitted by the limited liability constraint, therefore, the only bonus payment allocated at the breakthrough is given the agent who achieves it. $W_{i,0}$ denotes the transfer at time zero.

The principal seeks to maximize her payoff over bonus contracts and effort functions, solving the following program:

\begin{equation}
\max_{a_{i,t}, w_{i,t}, W_{i,0}} \int_0^\infty p_t a_{i,t} (\pi - w_{i,t}) e^{-\int_0^t (p_s a_s + r) ds} dt - W_{i,0}.
\label{OB}
\end{equation}

subject to

$$a_{i,t} \in \arg\max_{\tilde{a}_{i,t} \in [0,\bar{a}]} \int_0^\infty (p_t \tilde{a}_{i,t} w_{i,t} - \kappa \tilde{a}_{i,t}) e^{-\int_0^t (p_s (a_{-i,s} + \tilde{a}_{i,s}) + r) ds} dt.$$ \label{IC}

\begin{equation}
\int_0^\infty (p_t a_{i,t} w_{i,t} - \kappa a_{i,t}) e^{-\int_0^t (p_s a_s + r) ds} dt + W_{i,0} \geq \bar{U}.
\label{IR}
\end{equation}

\begin{equation}
w_{i,t} \geq 0, W_{i,0} \geq 0,
\label{LL}
\end{equation}

for $i \in \{1, 2\}$ and time $t$, where $a_s = \sum_j a_{j,s}$ and $a_{-i,s} = \sum_{j \neq i} a_{j,s}$.\textsuperscript{17}

The principal’s objective function (OB) is the expected payoff of the principal if each agent $i$ is paid $w_{i,t}$ if he obtains a breakthrough at time $t$ and his effort function is given by $a_{i,t}$. Since the effort of the agents is unobserved, the (IC) constraint says that the agent has to find it optimal to exert the effort that the principal wants to induce. Finally, the (IR) constraint says that the agents’ payments have to be greater than their outside option.\textsuperscript{18}

\textsuperscript{16} In the one task project there is only one possible history preceding a breakthrough, the history in which no breakthrough has occurred yet. Thus, the contract can condition only on the timing of the breakthrough and the agent who attained it.

\textsuperscript{17} $a_{-i,s} = 0$ if $n = 1$.

\textsuperscript{18} In the absence of a limited liability constraint, the principal can extract full surplus and implement the first best effort by “selling each agent his own arm”. That is, each agent pays the expected value of his experimentation and receives $\pi$ if he is first to attain a breakthrough.
3.1 Optimal contract

The principal designs the optimal bonus contract so as to pay as little rents as possible to the agents without giving them incentives to procrastinate. The optimal contract that induces effort functions \((a_{i,s})_{i=1}^n\) is characterized in the following proposition.

**Proposition 1** (Agent’s contract). Suppose the principal wants to implement effort functions \((a_{i,s})_{i=1}^n\) at minimum cost. Each agent i’s bonus wage \(w_{i,t}\) satisfies the following differential equation

\[
\dot{w}_{i,t} = (a_{-i,t} + r)(w_{i,t} - \kappa) - r\kappa e^{\gamma t},
\]

where \(x_t = \int_0^t (a_{i,s} + a_{-i,s}) \, ds + \log \left( \frac{1-p}{p} \right)\) and the boundary condition is \(w_{i,T_i} = \kappa(e^{\gamma T_i} + 1)\) with \(T_i = \sup\{t | a_{i,t} > 0\}\).\(^{19}\)

All proofs are in the appendix.

The differential equation (2) can be solved explicitly for any choice of effort functions \((a_{i,s})_{i=1}^n\).

Figure 1 illustrates how the bonus varies as function of the time at which the task is completed. An important observation is that when rewarded by this optimal contract the agents are indifferent between exerting effort now and at the next instant.\(^{20}\) Intuitively, if they had strict incentives to exert effort at some time the principal could lower the payment at that instant and reduce her costs. Changing the contract at one instant, however, can affect the incentives at all other times, not just at the next instant. In appendix A.3 we show, using Pontryagin’s principle, that the contract that satisfies these “local” incentive compatibility constraints is, in fact, globally incentive compatible.\(^{21}\)

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\(^{19}\) The presence of another agent only affects agent \(i\)’s contract via the function \(a_{-i}\). Thus, the Proposition also characterizes the optimal contract in settings with more than two agents, with asymmetric agents and in settings in which the agents do not necessarily observe their coworkers breakthroughs (as in Section 3.2). It is also possible to derive a PDE when the arms are not perfectly correlated using an analysis analogous to the one we outline in the appendix.

\(^{20}\) To gain intuition for the differential equation that characterizes the optimal bonus contract, consider the agent \(i\)’s decision of whether to shift an amount \(\varepsilon\) of effort from time interval \([t, t+dt]\) to time interval \([t+dt, t+2dt]\). The expected payoff of agent \(i\) at time \(t\) can be approximated as

\[
V_{i,t} = \left( w_{i,t} (1 - e^{-a_{i,p}dt}) - \kappa a_{i,t} dt \right) + e^{-(r+p(a_{i,t} + a_{-i,t}))dt} V_{i,t+dt},
\]

where \(e^{-a_{i,p}dt}\) is the probability that player \(i\) does not get breakthrough in instant \(dt\). Replacing \(V_{i,t+dt}\), approximating the exponentials with a second order Taylor series and computing \(\frac{\partial}{\partial (dt)^2} \left( \frac{\partial V_{i,t}}{\partial t} \right)\) and setting it to zero one obtains equation (2) (see appendix A.2 for a detailed derivation).

This derivation is closely related to the one in Bonatti and Hörner (2011). In their model agents are also indifferent between exerting effort in two consecutive instants. However, the reason why agents are indifferent is different in the two models. In their model the indifference arises because of the agents’ optimization problem, whereas in my model it is decided by the principal in order to minimize the cost of incentives for effort.

\(^{21}\) Halac et al. (2016b) find a similar result in a discrete time model.
The following proposition characterizes the optimal effort functions which together with Proposition 1 pin down the optimal contract, for \( n \in \{1, 2\} \) agents.

Define

\[
    w_i^*(T) = \kappa + \frac{1 - \bar{p}}{\bar{p}} \kappa \left( -e^{nt\bar{a}}r + e^{r(t-T)} + ((-1+n)t+T)\bar{a} \right) \frac{1}{r + \bar{a}},
\]

and

\[
    T^* = \frac{\ln \left( \frac{\pi - \kappa}{\kappa} \right) - \ln \left( \frac{1 - \bar{p}}{\bar{p}} \right)}{(1+n)\bar{a}} = \frac{n}{n+1} \bar{T}.
\]

where \( \bar{T} \) denotes the efficient experimentation threshold as shown in equation (1). The bonus wage \( w_i^*(T) \) is obtained by solving differential equation (2) assuming all agents exert maximum effort until some time \( T \).

Let \( T(\bar{U}) \) be the experimentation threshold that gives agents expected payoff \( \bar{U} \) under \( w_i^*(T(\bar{U})) \). \( T(\bar{U}) \) is the solution to

\[
    v(T(\bar{U})) = \frac{\kappa (1 - \bar{p}) \bar{a} e^{-rT(\bar{U})} \left( -e^{T(\bar{U})\bar{a}} \right) + \bar{a} \left( e^{rT(\bar{U})} - 1 \right) + r}{r (r - \bar{a})} = \bar{U}.
\]

Define

\[
    T^{**} = \max \{ \min \{ \bar{T}, T(\bar{U}) \}, T^* \},
\]

and

\[
    W_0 = \max \{ 0, \bar{U} - v(\bar{T}) \}.
\]

**Proposition 2 (Optimal contract).** The unique optimal bonus contract \( w_{i,t} \) is given by

\[
    w_{i,t} = w_i^*(T^{**}) \text{ for } t \leq T^{**} \text{ and } w_{i,t} = 0 \text{ for } t > T^{**}, \text{ and } W_{i,0} = W_0,
\]

with \( a_{i,t} = \bar{a} \) for \( t \leq T^{**} \) and \( a_{i,t} = 0 \) thereafter for each agent \( i \).

Proposition 2 shows that in the benchmark one-task case, the optimal contract for symmetric agents is symmetric and all agents work at maximum effort until a time threshold.\(^{22}\) Figure 1

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\(^{22}\)To gain intuition for why the principal sets the effort at the maximum until a deadline, let us consider the dynamic programming problem of the principal. Let \( \Pi_{i,t} \) and \( \bar{\Pi}_{i,t} \) denote the expected payoff that the principal and the planner obtain from agent \( i \)’s experimentation and let \( V_{i,t} \) denote the expected payoff of agent \( i \) at time \( t \). Note that \( \Pi_{i,t} = \bar{\Pi}_{i,t} - V_{i,t} \). Consider the principal’s decision to shift effort \( \varepsilon \) from time interval \([t, t+dt]\) to time interval \([t + dt, t + 2dt]\). To evaluate this trade-off we first write the value function of the principal as

\[
    \Pi_{i,t} = (1 - e^{-p(\bar{a} - a_{i})dt}) \Pi - \kappa a_{i,t} dt + e^{-(r + p(\bar{a} - a_{i})dt)} \Pi \Pi_{i,t+dt} - V_{i,t},
\]
illustrates how the optimal contract gives higher transfers and increases more slowly than the no-rent contract \( w_{t}^{NR} = \kappa / p_{t} \) for the same effort function. Intuitively, the optimal bonus payment increases in order to compensate the agents as they become more pessimistic over time but it cannot increase so fast as to make agents want to delay their effort. \( w_{t}^{*}(T^{*}) \) is the lowest bonus contract that provides incentives to exert maximal effort up to time \( T^{*} \), as the principal aims to minimize the information rents paid out to the agents.

Experimentation stops inefficiently early under the optimal contract unless the outside option is so high as to set \( T^{**} = \bar{T} \). Recall that efficiency requires that players experiment at their maximum effort until \( \bar{T} > T^{*} \) (see equation (1)). This inefficiency arises because agents have to be compensated with a higher bonus and, thus, higher procrastination rents if they are expected to experiment until a later time threshold (as seen in Figure 2). Thus, the principal trades off longer experimentation with increased rents and opts to stop experimentation at an inefficient level. Recall that, at the first best, experimentation stops when \( p_{t}\pi = \kappa \). When the belief is such that \( p_{t}\pi \) approaches \( \kappa \), the principal has to pay a bonus that is close to \( \pi \) in order to induce effort. Thus, it since with probability approximately \( (1 - e^{-\bar{a} \cdot dt}) \) there is a breakthrough in interval \([t, t + dt]\) and with probability \( e^{-\bar{a} \cdot (\bar{a} + a - i) dt} \) there is breakthrough in interval \([t, t + dt]\). The first order effect of shifting effort is zero. Approximating the exponentials with a second order Taylor expansion we obtain

\[
\sum_{k=1}^{\pi} \left. \frac{\partial \Pi_{k,t}/\partial \epsilon}{\partial (dt)^2} \right|_{dt=0} = - (p_{t}\pi - \kappa) r - \frac{\partial}{\partial (dt)^2} \left( \frac{\partial V_{i,t}}{\partial \epsilon} \right) < 0. \tag{6}
\]

The last term in the previous expression is zero because the agent is made indifferent between exerting effort in two consecutive instants under the optimal contract (see footnote 20). The first term is negative when \( p_{t}\pi > \kappa \)—which is true as long as experimentation is socially efficient. Details of these computations can be found in section A.2 of the appendix.

For comparison, in Bonatti and Hörner (2009) the payment is decreasing. Because the principal must compensate both agents when there is a success, the public-good rents (as introduced in the next section), are large enough that the optimal contract is decreasing in time. In Klein (2016) the optimal contract is also decreasing for small enough \( r \) because an impending deadline reduces the costs of providing incentives as time progresses.
cannot be optimal for the principal to have the agents work until time $\bar{T}$. By having the agents stop slightly earlier the principal incurs a loss in profits from experimentation of second order, since she is obtaining nearly no surplus from breakthroughs at times close to $\bar{T}$. At the same time, the principal sees a first order drop in the bonus at every time $t$ since $w^*_t$ is strictly increasing in $T^*$ for all $t < T^*$.

Thus, the principal gains from stopping experimentation strictly before time $\bar{T}$. Only when the agents’ outside option is large enough is experimentation efficient.

When the agent’s outside option binds, the principal improves the agent’s experimentation assignment so that the agent receives more information rents. When there are no gains from a longer experimentation assignment, because the agents are experimenting at the efficient level, the principal provides a lump-sum payment or signing bonus at time zero. The intuition for this result is the following. A lump-sum payment generates a cost to the principal. A longer assignment that generates the same cost also produces surplus through the agents’ experimentation. Therefore, the principal will choose the latter.

Let’s now turn to a descriptive analysis and comparative statics of the optimal contract.

**Corollary 1.** The bonus wage $w^*_t(T^*)$ is non-decreasing in $t$ and is strictly increasing in $t$ unless $r = 0$ and $n = 1$.

That is, when $r = 0$ and $n = 1$ the principal offers a constant bonus. In the absence of competition and discounting, an agent faces no cost from procrastination and the principal must offer a constant bonus. When there is discounting or competition from other agents, however, the principal can afford to pay the agents less when they are more optimistic about the feasibility of the task, as they expect the bonus with higher probability. The principal may save from offering an increasing prize that is sensitive to the perceived feasibility or difficulty of the innovation.

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$^{24}$The derivative of $w^*_t$ with respect to $T^*$ is given by $\kappa a e^{\beta((n-1)t+T^*)+r(t-T^*)+x_0} > 0$. 

Figure 2: Solid curves: bonus contracts for different stopping times. Parameter values: $(\kappa, \bar{a}, \bar{p}, \pi, r, \bar{U}) = (1/4, 1.9/10, 1, 0.5, 0)$. Dashed curve: no-rent bonus payment. The agents’ bonus contracts increase in the experimentation threshold.
Increasing the number of agents from 1 to 2 reduces the bonus at every \( t \) and extends the experimentation threshold. Thus, the principal distorts the optimal contract less when contracting with more agents. In the appendix we provide comparative statics on the number of agents for \( n \geq 1 \). We show that as the number of agents \( n \) increases, the stopping time \( T^* \) converges to the efficient stopping time \( \bar{T} \). When the outside option is zero, the payoff of the principal increases as the agents receive less information rents and the aggregate effort that the agents may exert increases. Moreover, even keeping total capacity fixed, that is keeping \( n\bar{a} \) constant, the principal prefers to hire more and more agents. When the outside option, \( \bar{U} \), is greater than zero there is a maximum number of agents the principal hires.

When \( \bar{U} = 0 \) the principal prefers to split capacity into more and more agents since agents create an externality on each other that reduces the procrastination rents. First, if an agent stops working it is likely that another agent gets the reward. Second, agents procrastinate in order to put a wedge between their private belief and the principal’s and exert effort when it is most profitable. The smaller the share of the total effort each agent represents, the less control each agent has over the gap between beliefs and the less he stands to gain from procrastination. Thus, as the number of agents increases, procrastination becomes less profitable.

**Corollary 2** (Comparative statics). *Suppose the IR constraint does not bind, then the optimal payment scheme has the following properties:*

1. \( w_t^* \) is decreasing in \( r \) and in \( \bar{p} \) and increasing in \( \bar{a} \).

2. *The terminal belief and the total experimentation conditional on no breakthrough do not depend on \( \bar{a} \) or \( r \). The terminal belief increases in \( \kappa \) and decreases in \( \bar{p} \).*

Bonus contracts are decreasing in \( r \). As agents become more impatient they value future bonuses less and thus their temptation to procrastinate is reduced. The bonus is increasing in each agent’s capacity, \( \bar{a} \). A greater capacity allows the agents to learn more about the feasibility of the project and, thus, they must be compensated with more information rents.

The total experimentation conditional on no breakthrough is given by \( T^*\bar{a}n \) and does not depend on \( \bar{a} \) nor \( r \), nor does the terminal belief. The principal chooses a terminal belief that only depends on the benefits of the project, its prior probability of being good and the number of agents.

### 3.2 Optimal disclosure of breakthroughs

We have seen that when breakthroughs are observed the optimal contract is a contest in which only the first worker who succeeds in producing a breakthrough receives a reward. We now ask whether the principal can gain from letting the agents work independently and rewarding them for
successes even after other workers have completed the task. More generally, we ask whether the
principal can design an information disclosure policy to his advantage so as to reduce the costs of
inducing effort.

Suppose that breakthroughs are observed by the principal and the player who attained them but
not by the opposing player. Would the principal choose to disclose a breakthrough as soon as it
occurs or would she choose to delay its disclosure? If the breakthrough is disclosed immediately
the principal avoids costly duplicated effort. If she delays disclosure or does not disclose, and
rewards agents who obtain breakthroughs after the first breakthrough has been attained, the agents’
beliefs that they’ll be rewarded falls more slowly.\(^{25}\) As a result the principal can offer lower
bonuses.

Surprisingly, we find that, as stated in Proposition 3 below, the principal cannot gain from not
disclosing a breakthrough. The intuition for this result is the following. At the optimal contract,
which minimizes procrastination rents, the constraint that binds is that the agent does not wish to
delay effort from one instant to the next. In continuous time, these binding incentive constraints
imply that the instantaneous expected payment satisfies a differential equation. This differential
equation together with the boundary condition completely determine the instantaneous expected
payoff. In particular, the expected payment is independent of the presence of other workers. This
means that the bonuses that are paid upon success depend on the opposing players’ experimentation
in such a way that the expected payment does not. Because only the expected payment enters the
utility of the principal and the agent, the cost of providing incentives to each agent is independent
of the presence of other workers. This implies that the principal does not gain from delaying
disclosure as, even though the bonuses are reduced, the expected payments to agents are not.\(^{26}\)

**Proposition 3 (Optimal disclosure).** *If disclosures of breakthroughs are verifiable, in the optimal
bonus contract and disclosure policy the principal pays the bonus \(w_{i,t}\) defined in Proposition 1 and
discloses it immediately.*\(^{27}\)

\(^{25}\)The bonus increases in the “rate of disclosure” of the opponent’s breakthrough, as shown in the proof of Proposition 3.

\(^{26}\)Note that if we set \(\hat{v}_{i,t}a_{i,t} = a_{i,t}(p_{t}w_{i,t} - \kappa)e^{-\int_{0}^{t} p_{s}a_{i,s}ds - rt}\), the integrand of the agent \(i\)’s utility, then the derivative
of \(\hat{v}_{i,t}\) with respect to \(a_{i,t}\) only depends on \(a_{-i,t}\) through \(\hat{v}_{i,t}\). Therefore, the ODE implied from the requirement that
the agent does not shift effort from an instant to the next does not depend on \(a_{-i,t}\). In fact, one can show that this ODE
is given by \(\dot{\hat{v}}_{i,t} = -\hat{v}_{i,t}a_{i,t} - (1 - \hat{p})\kappa e^{-rt}a_{i,t}\). Thus, by Picard’s existence theorem \(\hat{v}_{i,t}\) is uniquely determined by its
boundary condition \(\hat{v}_{i,T_{i}} = 0\) and its value does not depend on agent \(-i\)’s effort. In this case this property stems from
the separability of \(i\)’s and \(-i\)’s efforts in \(i\)’s instantaneous utility.

\(^{27}\)Because we assume that disclosures are verifiable, we restrict attention to disclosure policies in which the prin-
cipal fully reveals that a breakthrough has occurred but may delay this disclosure. A partial disclosure policy is for
instance one in which the principal flips a coin at some time and sends a signal in the event that either the flip is heads
or there was a breakthrough. In this case, conditional on a signal the belief that the opponent has had a success would
rise but not to one. More generally, the principal could commit to a probabilistic disclosure policy as in Kamenica and
Gentzkow (2011).
This result demonstrates that the optimal contract is a winner-take-all: the first player to achieve a success is the only one who receives a reward.

Our result complements the analysis in Halac et al. (2016b). In their model, the contest designer has a budget constraint and, therefore, the problem of a principal who has a fixed maximum prize rather than the goal of minimizing expected payouts for a given level of experimentation. They find that a principal would optimally delay disclosure of breakthroughs in some situations.

In section A.6 of the appendix we show that it is unprofitable for an agent to hide a breakthrough. Even though the bonuses increase over time, they do not increase fast enough so as to overcome discounting and competition.

4 Project with two tasks

In a project with two tasks the agents have to obtain a success in the first task before they can start experimentation in the second task. If they achieve a breakthrough in the second task they complete the project. There are many real-world examples in which innovation involves the completion of uncertain and sequential tasks. Consider, for example, a group of engineers developing software. Steps in the production process involve planning the functionalities and characteristics of the product, creating a prototype, improving performance and solving issues and bugs. If any of these steps cannot be completed satisfactorily the project may be abandoned. Medical researchers working on the development of a drug must first find a promising class of compounds and experiment to find a drug that has the desired effect. In research contexts an important discovery leads to new avenues of research that build on it.

If there are two contracts that give the same discounted payoffs after every history—and, thus, implement the same effort—we assume without loss that the principal chooses the contract that pays each agent at the earliest possible time. For this choice of contract, in analogy to the one-task case, it is not profitable for the principal to reward an agent when another agent succeeds.

4.1 The second task contract

A key step that allows us to solve the two-task model is characterizing the continuation contract after any given first-task history. In the following Proposition, we show that the second-task con-

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28 The latter problem is the dual of the problem we consider.

29 When analyzing the difference between the full-disclosure contract and the no-disclosure contract, which correspond to the winner-takes-all and the hidden equal-share in Halac, Kartik, and Liu (2016a), the use of a time-dependent prize becomes relevant. From equation (2) in Proposition 1, the optimal contract in the hidden equal-share setting when \( r = 0 \) is a time independent prize (as \( a_{-t} = 0 \) in that case) while it is not constant in the winner-takes-all setting (as \( a_{-t} > 0 \) in equation (2)). Thus, time-independent prizes make the hidden equal-share contract more appealing relative to the winner-takes-all.
tract has the same form as the one-task contract, with agents exerting maximal effort, but with experimentation deadlines that depend on the realized first-task history.

Define

$$w_{i,t}^2(T^2_i, \tau) = \begin{cases} 
\kappa^2 + e^{\int_t^{T^2_i} e^{-r\int_t^s a_{1,i}^2 ds + x_0^2 r\kappa^2 dl + \kappa^2 e^{-\int_t^s (r-a^2_i) ds + x_0^2 r\kappa^2 dl}}} & \text{if } \tau \leq t \leq T^2_i, \\
0 & \text{if } t > T^2_i.
\end{cases}$$

(7)

where $x_0^2 = \log \left( \frac{1-p_0^2}{p_0^2} \right)$. Note that $w_{i,t-\tau}^2(T^2_i, \tau)$ solves differential equation (2), which characterizes the optimal bonus contract in the one-task project, given effort functions $(a_{i,t-\tau})_{t \geq \tau}$ and experimentation deadlines $T^2_i$, for each agent $i$.

**Proposition 4 (Second-task contract).** Consider a history in which agent $k'$ achieved the first-task breakthrough at time $\tau$. There are experimentation thresholds $T^2_i(k', \tau)$ for $i \in \{1, 2\}$ such that $a_{i,t}^2 = \bar{a}$ if $\tau \leq t \leq T^2_i(k', \tau)$ and $a_{i,t}^2 = 0$ if $t > T^2_i(k', \tau)$. Agent $i$ receives bonus $w_{i,t}^2(T^2_i(k', \tau), \tau)$, as defined in equation (7), if he succeeds in the second task at time $t \in (\tau, T^2_i(k', \tau))$.

Proposition 4 shows that in the second task the agents work at the maximum effort until a threshold time and that the principal offers a contract analogous to the optimal single-task contract. Thus, although the set of feasible contracts is large, the second-task optimal contract for agent $i$ after a history can be parametrized by a single variable: $i$’s experimentation threshold in the second task, $T^2_i(k', \tau)$. This result allows us to write the two-task problem as an optimal control problem in which the second-task experimentation threshold is a control variable. To establish the result in Proposition 4, we show that the optimal way to provide a given promised utility after the first breakthrough is realized involves a non-negative bonus at the beginning of the second task and a second-task bonus contract that is analogous to the one-task contract.

### 4.2 The first task contract

The expected payoff of the agents after realization of the first-task breakthrough has two components: the bonuses that they receive at the time of the first breakthrough and the expected payoff they receive from their experimentation assignment in the second task. By the same argument as in the single-task case, the principal does not provide bonuses at the completion of the first task to the agents who do not succeed. However, an agent’s expected payoff following another agent’s success is not necessarily zero. Rather, it is strictly positive if he receives an experimentation assignment involving positive effort in the second task.

Let $w_{i,t}^{1,j}$ denote the bonus that agent $j$ receives when he achieves a breakthrough in the first task at time $t$ and let $v_{i,t}^{1,j}$ denote the expected payoff that agent $i$ obtains in the second task after that history. $v_{i,t}^{1,j}$ consists of the procrastination rents that $i$ obtains from experimentation in the second
task. Let \( T^2(j, \tau) = (T^2_i(j, \tau))_{i=1}^n \) be a vector of second task experimentation thresholds. From the characterization in Proposition 4, \( v_{i,t}^j \) is given by

\[
v_{i,t}^j = \frac{e^{-\tau T^2(i,t)} \kappa^2 \left( r - e^{\tau T^2(i,t)} \bar{a}_t + \left( -1 + e^{\tau T^2(i,t)} \right) \bar{a} \right)}{r(r - \bar{a})} (1 - \bar{p}). \quad (8)
\]

The expected payoff \( v_{i,t}^j \) depends only on \( i \)'s experimentation thresholds and not on an opposing player's.\(^{30}\) We will denote it \( v_{i,t}^j(T^2_i(j,t)) \) when we want to make this dependence explicit.

Let \( y_i^1 = \int_0^\infty a_i^1 ds \), \( x_i^1 = \log \left( \frac{1}{1 + \frac{p_i}{\bar{p}_i}} \right) \), \( x_i^1 = x_0 + y_i^1 \) and define \( u_{i,t} = \tilde{w}_{i,t}^1 + v_{i,t}^j \). \( u_{i,t} \) is the total reward that agent \( i \) receives when he succeeds in the first task at time \( t \). It consists of \( i \)'s bonus and the expected payoff from his experimentation assignment in the second task. The following Proposition characterizes the minimum reward that an agent needs to receive for him to choose a given effort schedule given the effort schedules of the other agents.

**Proposition 5** (Minimum first-task reward). The minimum reward, \( u_{i,t}^{\text{min}} \), that each agent \( i \) must receive in order to implement effort schedules \( \{(a_{j,t})_{t \geq 0}\}_{j=1}^n \) must satisfy the following differential equation:

\[
u_{i,t}^{\text{min}} = (u_{i,t}^{\text{min}} - \kappa^1_i) (a_{-i,t}^1 + r) - \sum_{j \neq i} v_{i,t}^j a_{j,t}^1 - \kappa^1_i r e^{x_i^1}. \quad (9)
\]

with transversality condition

\[
u_{i,t}^{\text{min}} = \kappa^1_i \left( 1 + e^{x_i^1} \right) - \int_{T_i^1}^{\infty} \sum_{j \neq i} v_{i,t}^j a_{j,t}^1 e^{-s_1 a_{j,t}^1 ds} - r(t - T_i^1) dt.
\]

where \( T_i^1 = \sup\{t | a_{i,t}^1 > 0\}. \quad (31)\)

From Proposition 5 we see that agent \( i \)'s expected payoff at the contract that minimizes expected bonuses depends on the payoff that \( i \) obtains from experimentation in the second task when agent \( j \neq i \) succeeds, \( v_{i,t}^j \). As in the one-task case the intuition for this functional form for the expected payoff is that \( u_{i,t}^{\text{min}} \) makes agent \( i \) exactly indifferent between exerting effort in one instant and the next.

Let \( a_i^1 = \left((a_{j,t}^1)_{t \geq 0}\right)_{i=1}^n \) be a vector that contains the first-task effort functions of all agents. Let \( w_{i,t}^1(a^1) \) denote the optimal bonus contract offered to agent \( i \) in a single task project in which the implemented efforts are summarized in \( a^1 \). From Proposition 1, \( w_{i,t}^1(a^1) \) is characterized by

\(^{30}\)This observation is useful for our results on information disclosure in section 4.3

\(^{31}\)\( \sum_{j \neq i} v_{i,t}^j = 0 \) if \( n = 1 \).
Solving the differential equation (9) for $u_{i,t}^{\min}$ we obtain,

$$u_{i,t}^{\min}(a^1) = w_{i,t}^1(a^1) + \sum_{j \neq i} \int_{t}^{\infty} e^{-\int_{t}^{\tau}(r + a_{j,t}^1)ds} a_{j,\tau}^1 v_{i,\tau}^j d\tau.$$

Agent $i$’s payoff consists of two terms. The first term is the bonus wage of a one-task contract—which contains all the necessary procrastination rents. The second term is the payoff that $i$ would obtain if he were to stop exerting effort in the first task and then work in the second task according to the equilibrium strategies after some other agent completes the first task. The second term is positive whenever agent $i$ receives a positive experimentation assignment in the second task after another agent’s success. That is, whenever agent $i$’s expected payoff is strictly positive after any other agent achieves a breakthrough he must receive—in addition to the procrastination rents—a compensation that prevents him from shirking in the first task. Thus, the fact that the agents are able to obtain information rents in the following task generates an endogenous free-riding concern. We will see that in equilibrium the agents do not free-ride on the other agent’s work because their compensation is raised to prevent it. These rents are not present in a one task project or in the last task of a two-task project.

The analysis now divides into cases based on whether limited liability or incentive compatibility binds in setting the value to agents of completing the first task. Recall that $u_{i,t}^{\min}$ minimizes the expected bonuses for the completion of the first task, subject to incentive compatibility. Agent $i$ may need to receive a higher payoff, $u_{i,t}$, in the optimal contract when he achieves a success at time $t$, however, as the bonuses implied by $u_{i,t}^{\min}$ may violate the limited liability constraint. In

$$32\text{In the notation of the two-task case } w_{i,t}^1(a^1) = \kappa^1 + e^{\int_{t}^{T^1} e^{-\int_{t}^{s}(r + a_{i,s}^1)ds + x_0} r\kappa^1 ds + \kappa^1 e^{-\int_{t}^{T^1}(r - a_{i,s}^1)ds + x_0} dr}.$$  

where $x_i^1 = \log \left(\frac{1 - p_i^1}{p_i^1}\right)$ and $T_k^1 = \sup\{t|a_{i,k}^1 > 0\}$ for $k \in \{1, 2\}$. This bonus contract is analogous to the one defined by equation (7).

$$33\text{To gain intuition for why these rents occur, we refer to the dynamic programming heuristic. Consider the decision of the agent to shift effort } \varepsilon \text{ from time interval } [t, t + dt] \text{ to time interval } [t + dt, t + 2dt]. \text{ The expected payoff of agent } i \text{ at time } t \text{ satisfies}$$

$$V_{i,t} = \left(u_{i,t}(1 - e^{-\int_{t}^{T^1} a_{i,s}^1 ds}) - \kappa a_{i,t}^1\right) + e^{-\int_{t}^{T^1}(r + a_{i,s}^1 + \kappa)ds} V_{i,t+dt} + v_{i,t}(T_i - i, t))(1 - e^{-\int_{t}^{T^1} a_{i,s}^1 ds}).$$

Approximating the exponential with a second order Taylor expansion we obtain

$$\frac{\partial}{\partial(t)^2} \left(\frac{\partial V_{i,t}}{\partial \varepsilon}\right) = \dot{u}_{i,t} - (a_{i,t}^1 + \kappa)(b_{i,t} - \kappa) + r ve^\varepsilon + p_i^1 v_{i,t}(T_i - i, t) a_{i,t}^1. \quad (10)$$

The last term in 10 is positive as long as $i$ exerts effort and $i$ exerts effort in the second task. Thus, if the principal offers expected payoff $u_{i,t} = w_{i,t}$, the first three terms sum to zero, and agent $i$ has incentives to shift effort to the future. In the two task case, agents get a positive surplus in the second task because they are given procrastination rents. If the principal were to only give them an expected payoff equal to the bonus wage in the one-task case the agents would prefer to not work for an instant and let the other agents achieve the first discovery.
fact, suppose that the principal implements effort functions $a^1$ and second task experimentation thresholds $T^2(k, \tau)$. If $u_{i,t}^{\text{min}}$, defined in (11), is strictly less than agent $i$’s payoff from second-task experimentation, $v_i^t$, for all $t$, we have that the first-task bonus implied by $u_{i,t}^{\text{min}}$, $\tilde{w}_{i,t}^{1,\text{min}} = u_{i,t}^{\text{min}} - v_i^t$, is strictly negative. Since the limited liability constraint restricts the bonus to be positive, the actual reward at time $t$, $u_{i,t}$, must be strictly above $u_{i,t}^{\text{min}}$.

In the case in which the bonus implied by $u_{i,t}^{\text{min}}$, $\tilde{w}_{i,t}^{1,\text{min}}$, is strictly positive for all $t$, the payoff in the second task is not sufficient to provide incentives in the first task and $u_{i,t}^{\text{min}} = u_{i,t}$. We refer to this case as the **costly incentives case**. In contrast, when $u_{i,t}$ does not satisfy the differential equation (9) at any time $t$, the expected payoff in the second task provides enough incentives for effort in the first task. We refer to this case as the **cheap incentives case**. When the parameters do not fall in the cheap or costly incentives case we’ll say they fall in the **intermediate incentives costs case**.

Propositions 6 and 7 below characterize the efforts that are optimal in the costly and cheap incentives cases with two agents, respectively. These efforts can be determined from the primitives. The parameters of the problem fall in the costly incentives case if $u_{i,t}^{\text{min}}(a^{1,co}) > v_i^t(T_i^{2,co}(i,t))$ where $a^{1,co}$ denotes the optimal vector of efforts and $T_i^{2,co}(i,t)$ denotes the optimal second-task experimentation threshold of the cheap incentives case. If under the cheap-incentives-case optimal effort vector, $a^{1,ch}$, and second task threshold, $T_i^{2,ch}(i,t)$, the inequality $u_{i,t}^{\text{min}}(a^{1,ch}) < v_i^t(T_i^{2,ch}(i,t))$ holds, then we are dealing with the cheap incentives case. Otherwise the problem falls in the intermediate incentives cost case.

Roughly speaking, the problem is more likely to fall in the costly incentives case if $\kappa^1$ is large relative to $\kappa^2$, when $r$ is large (i.e. the value of the second task experimentation is small) and when $\pi^1$ is large (which implies $T_i^1$ and $T_i^{1,j}$ are large). For example, from equation (8), if $r \geq 2\bar{a}$ and $\kappa^2 \leq \kappa^1$ the problem immediately falls in the cheap incentives case. These conditions guarantee that the procrastination and public-good rents in the first task are large relative to the procrastination rents in the second task. The opposite relationships on the primitives make the problem more likely to fall in the cheap incentives case.\footnote{Condition 1 characterizes the primitive conditions for which the problem falls in the cheap incentives case.}

In what follows we focus on the case with two agents which has the richest dynamics.

### 4.2.1 Costly first-task incentives

In the costly first-task incentives case the minimum first-task reward binds and we have $u_{i,t} = u_{i,t}^{\text{min}}$ as defined in (11). From the discussion above, in this case $v_i^t < u_{i,t}$ and the successful agent receives a bonus upon the completion of the first task. The following proposition characterizes the effort schedules that the principal implements in both tasks. Let $V_a = \left(-x_0^2 + \log \left( \frac{2x_0^2 - \kappa^2}{\kappa^2} \right) \right)$, $V_{i,t}$
is the total amount of experimentation when the agents stop at the efficient belief in the second
---that is, the belief such that the expected payoff from a success equals the cost of effort.

**Proposition 6** (Costly first-task incentives). In the costly first-task incentives case, agents exert
maximum effort in the first task until thresholds \((T^1_i, T^{-1}_i)\). If agent \(i\) achieves the first breakthrough
at time \(t\), agent \(-i\)'s experimentation threshold, \(T^2_{-i}(i, t)\), decreases in the timing of the first breakthrough, and agent \(i\) experiments until time efficient stopping time \(T^2_i(i, t) = V_a/\bar{a} - T^2_{-i}(i, t)\). If \(W_{i,0} > 0\) then \(i\) stops experimentation at the efficient threshold in each task after every history.\(^{35}\) When the IR constraint binds, the thresholds \(T^1_i\) and \(T^2_{-i}(i, t)\) are strictly increasing in the outside option \(\bar{U}\). The difference in the assignments of successful and unsuccessful agents decreases in the outside option.\(^{36}\)

From the Proposition, a successful agent is rewarded with a longer experimentation time, with competition that stops experimentation earlier, and with a larger bonus for a second-task breakthrough. Whereas if agent \(-i\) does not succeed in the first task, while his co-worker, agent \(i\), succeeds at time \(t\), \(-i\) is assigned experimentation threshold \(T^2_{-i}(i, t)\) which is decreasing in the total effort exerted up to time \(t\) by \(i\), \(\int_0^t a_{-i,s} \, ds\), and converges to zero as \(\int_0^t a_{-i,s} \, ds\) converges to \(\infty\).\(^{37}\)

An important implication of the characterization in Proposition 6 is that the principal prefers to
reward an agent via assignments rather than bonuses or time-zero lump-sum transfers. A successful agent receives an efficient experimentation assignment in the second task. Alternatively, the principal could have given the agent the same reward by giving him a larger bonus and reducing experimentation to a more profitable level, such as the implementing the one-task optimal threshold. However, this is not the optimal way to reward the agent. The reason is that an assignment that gives the agent the same payoff as a bonus also generates additional surplus resulting from the agent’s work. Thus, the principal prefers to reward the agent by “undistorting” his allocation. The same intuition explains why a higher outside option translates into longer experimentation assignments in the first task and in the second task for the unsuccessful agent. The agent receives a lump-sum transfer at time zero only if his experimentation thresholds are efficient after every history.

\(^{35}\)As shown in section 4.2.1 of the appendix. \(T_{-i}(i, t)\) solves \(M_i(T^2_{-i}(i, t)) = 0\) where

\[
M_i(\tau) = \left( (\zeta_i - 1) e^\int_0^\tau \bar{a}_{-i,s} \, ds \left( e^{\bar{a}_i - 1} - 1 \right) \kappa^2 (1 - \bar{\rho}^2) + \bar{\rho}^2 (\pi^2 - \kappa^2) e^{-2\bar{a} \tau} - \kappa^2 (1 - \bar{\rho}^2) \right)
\]

\[= -\bar{\rho}^2 \bar{a} \int_{\tau}^{V_a/\bar{a}} \left( \pi^2 - \kappa^2 \right) e^{-\bar{a} \tau - \bar{a} s - \bar{r} (s - \tau)} \, ds, \tag{12}\]

with \(\zeta_i\) a multiplier associated to the IR constraint.

\(^{36}\)The contract is uniquely optimal among bonus contracts. By risk neutrality, there are equivalent contracts that induce the same effort and give the same expected payoff after every history but provide payments at different times.

\(^{37}\)This follows from condition (12).
When the value of the outside option increases the agents receive a persistently better contract. Their thresholds in the first and second task are longer. The difference in the assignments of successful and unsuccessful agents decreases. Thus, the gains from a higher outside option persist beyond receiving a higher signing bonus. They translate into higher chances of success, better rewards for success and better terms of employment in both tasks in every eventuality. It is an empirical finding that the initial conditions of the labor market for generations of workers have persistent effects. That is to say, workers who face better labor market conditions when they graduate tend to experience persistently better outcomes throughout their careers.\textsuperscript{38} The presence of information rents, such as the procrastination rents that we describe, may provide an explanation for these empirical facts. Workers who graduate when the economy is good would receive allocations that are less distorted in all periods.

Furthermore, if a job after a promotion involves more private learning, and thus more rents, the surplus generated from allocating more responsibility to the employee makes the promotion preferable to a bonus as a reward. This observation provides an intuition for why it is commonly observed that firms use job assignments or promotions to reward workers instead of only monetary bonuses.\textsuperscript{39}

From Proposition 6 we see that the agent who succeeds is assigned an efficient second-task experimentation threshold, while the unsuccessful agent sees his experimentation threshold distorted down. The distortion increases in the effort that he exerts up to the time in which the breakthrough arrives. When the IR constraint does not bind, the experimentation threshold of the unsuccessful agent is even shorter than the (also inefficient) one-task threshold $T^*$. To gain some intuition for

\textsuperscript{38}See Beaudry and DiNardo (1991) and Baker et al. (1994) and Kahn (2010) for evidence on persistent effects of labor market conditions.

\textsuperscript{39}See, for example, Baker et al. (1988) and Gibbons and Waldman (1999) for a discussion of the bonuses versus promotions issue.
Figure 4: Experimentation stopping time in the second task of non-successful agent conditional on the timing of the first discovery. Parameter values: \((\kappa_1, \kappa, \bar{a}, \bar{p}_1, \bar{p}, \pi, \pi_1, n, \bar{U}) = (1/4, 1/4, 1, 9/10, 9/10, 5, 0, 2, 0)\).

this result note that the principal faces a tradeoff between letting the unsuccessful agent work up to her desired amount of experimentation—the optimal stopping time given that the successful agent experiments until the efficient belief—and decreasing the agents’ public good rents. The principal opts to distort the amount of experimentation down from her desired amount in the second task in order to reduce these rents. Reducing experimentation from the optimum generates a second order loss—due to optimality—while reducing the bonus produces a first order gain. Figure 4 illustrates how the experimentation threshold of the unsuccessful agent varies as function of the timing of the first breakthrough. The distortion increases in \(\int_0^t a_{-i,s} ds\) due to the principal’s discounting of the second-task payoffs and due to the fact that agents who slack expect the first breakthrough to arrive relatively later as they are not exerting effort.

The unsuccessful agent’s second-task experimentation threshold is decreasing in the time of the first breakthrough. Thus, the successful agent’s second-task expected payoff is increasing in the time of the first breakthrough. He is assigned a lengthier experimentation period in the second task and, as a result, receives more information rents. Figure 3 (right) shows the expected second-task payoffs for successful and unsuccessful agents. The successful agent’s overall payoff, however, considering the bonus, may increase or decrease in the timing of the first breakthrough. Thus, unlike in the one-task case, the bonus of the agent who succeeds may increase or decrease in the timing in which the first breakthrough arrives. The reason is that public-good rents decrease in the timing of the first breakthrough. If these rents dominate the procrastination rents, the expected reward increases in the timing of the first breakthrough. Figure 5 shows an example in which \(u_{i,t}\) is non monotonic.

Another surprising feature of the optimal contract is that even though the agents are known to be symmetric, their first task experimentation thresholds, \(T_{1i}^1\) and \(T_{2i}^1\), may differ. Even if the agents are identical it is sometimes optimal to offer them asymmetric experimentation assignments. One agent may receive a shorter experimentation assignment—and, thus, less information rents— or
may be barred from participating in the first task altogether. The asymmetries arise because of the public-good rents. When one agent is scheduled to work more he has lower incentives to free-ride. When the public-good rents are large relative to the competition between agents, the principal may prefer to let an agent sit idle instead of allowing him work and create an externality on his fellow worker by tempting him to slack. This is an example for why similar workers (or types of workers) may receive different job assignments with different career prospects.

Figure 5 (left panel) shows an example in which the experimentation thresholds are asymmetric. When $-\pi_1$, the cost to the principal of beginning stage 2, is small the optimal experimentation thresholds are symmetric. As $-\pi_1$ increases, agent $-i$ is allocated a shorter experimentation assignment. For larger $-\pi_1$ agent $-i$ receives no experimentation assignment. Intuitively, when the transfer after the first breakthrough is very negative, the value of beginning the second task is high and the value of $i$’s work in the first task is not high enough to justify the public-good rents that $i$ must receive when $-i$ is assigned much work. It may be optimal to have player $i$ work longer in the first task. In section (B.8) of the appendix we give a sufficient condition for an asymmetric contract. We show that the condition is satisfied when the first task is relatively safer ($x_i^1$ is small) and $\pi_1$ is low—that is, when its prior probability of being good is higher and the cost of initiating the second task is high. Having more agents reduces the incentives to procrastinate—because of competition—but increases the incentive to free-ride. This incentive to free ride is higher at the beginning of the first task because the second task allocation of the unsuccessful agent is distorted more as agents experiment without success in the first task. Reducing $\pi_1$ reduces the length of the first-task experimentation and, therefore, the temptation to free-ride is relatively high thought this experimentation. Thus, when the first arm has a high probability of being of good quality and the cost of starting the second task is high, each agent is less able to affect his private belief about the task by choice of effort and, therefore, procrastination is less of a concern than free-riding.

Define $u^\text{min}_{i,t}$ as in equation (11) with $v_{i,t}^i = v_{i,t}^i \left(T_{i}^2(i,t)\right)$. For the principal’s problem to fall in the costly first-task incentives case we must have $u^\text{min}_{i,t} > v_{i,t}^i \left(T_{i}^2(i,t)\right)$. One can verify whether this inequality holds, as a function of primitives, using the characterization in Proposition 6. Intuitively, the inequality will hold when the information rents in the second task are small relative to the first task, e.g. if the second task is nearly certain to be good, less risky and the marginal cost of effort is not greater than the first-task cost.

As explained, the principal distorts the agents’ second-task contracts. A natural question to ask is whether the principal would be better off hiring new agents for the second task. If the principal has access to identical agents, but only a fixed number of positions, she would fire and replace all the agents that don’t achieve a breakthrough in the first task and keep only the agent who succeeds. This result is stated in the following Corollary.

**Corollary 3 (Up or out).** If the principal could costlessly replace some agents with identical ones
in the second task, she would keep the agent who succeeds in the first task and replace the agent who does not. In the second stage, the agent who was present in the first task works until a longer time threshold.

On the other hand, if the principal has access to an additional pool of agents in the final stage, and is given the option to either replace or add more agents, she would choose to not replace any agents and add as many agents as possible. This result follows from the analysis of the one-task project: increasing the number of agents in the second task reduces the expected time at which the second-task breakthrough arrives, reduces the information rents in the second task and, reduces the value of free-riding in the first task. All of these mechanisms reduce the cost of providing incentives in both tasks.

The second-task experimentation threshold of the unsuccessful agent decreases monotonically to zero as a function of the effort exerted by the agent at the time of the first-task breakthrough. Thus, the value of the unsuccessful agent’s second-task experimentation decreases to zero in the time of the first discovery, provided that he exerts effort at that time. After some histories, it may be profitable for the principal to allocate the agent to an alternative task that provides less rents to the agent, even if this task is not very valuable ex-ante. Let task 2’ be identical to task 2 except that it gives transfer \( \tilde{\pi}_2 \) when completed and has prior probability of being good given by \( \tilde{p}_2 \).

**Corollary 4** (Demotion). *For every \( \tilde{\pi}_2 < \pi_2 \), there is a time \( \bar{t} \) and \( \tilde{p}_2 > \bar{p}_2 \) such that if agent \( i \) works at time \( t > \bar{t} \) and agent \( -i \) completes the first task at time \( t \), then the principal assigns agent \( i \) to task 2’.*

For small enough \( \tilde{\pi}_2 \) a single-task project consisting of task 2’ provides less payoff than a
single-task project consisting of task 2.\textsuperscript{40} Therefore, task 2’ would not be pursued by the principal in the absence of the first task. It only becomes useful as a punishment for previous bad performance.

The time at which the first milestone is achieved is bounded by the first task experimentation thresholds. These thresholds increase in the probability that the first task is good, \(\tilde{p}^1\). For \(\tilde{p}^1\) sufficiently close to one unsuccessful agents are assigned to safe but relatively unprofitable tasks after some histories.

4.2.2 Cheap first task incentives

We now turn to the case in which the rents from the second task are sufficient to provide incentives in the first task. When this occurs, the principal does not reward the agents for first-task successes. The second-task assignments correspond to the optimal one-task project assignments. The players experiment at maximum effort until the time threshold that maximizes the principal’s profits when task two is treated as a one task project. Unlike in the costly incentives case, experimentation assignments are always symmetric.

The agents’ payoffs do not depend in any way on the first-task history and, therefore, the principal does not need to observe the first task breakthrough in order to design the optimal contract. No agents are kept from participating in the first task. The optimal contract is characterized in the following proposition.

Let

\[
T_2^* = \ln \left( \frac{\pi^2 - \kappa^2}{\kappa^2} \right) - \ln \left( \frac{1 - \hat{p}^2}{\hat{p}^2} \right), \quad T_2 = \ln \left( \frac{\pi^2 - \kappa^2}{\kappa^2} \right) - \ln \left( \frac{1 - \tilde{p}^2}{\tilde{p}^2} \right),
\]

\[T_1(T_2) = \frac{\ln \left( \frac{\Pi^2(T_2) - \kappa^1}{\kappa^1} \right) - \ln \left( \frac{1 - \tilde{p}^1}{\tilde{p}^1} \right)}{2 \bar{a}},\]

\[
\int_0^{T_1(T_2)} \left( p_1 v_{i,t}(T_2(U)) - \kappa^1 \right) 2 \bar{a} e^{-\int_0^t \hat{p}_2 \dd \tau} \, \dd \tau = U,
\]

\[\int_0^{T_2^*} \left( p_1 v_{i,t}(T_2(U)) - \kappa^1 \right) 2 \bar{a} e^{-\int_0^t \hat{p}_2 \dd \tau} \, \dd \tau = \tilde{U},
\]

\[\text{where } \Pi^2(T) \text{ is the expected payoff the principal receives when both agents work until threshold } T_2 \text{ in the second task. } T_1(T_2) \text{ is the efficient experimentation threshold for a project that produces a transfer } \Pi^2(T_2) \text{ when successful.}

\text{Let } T_2(U) = (T_2(U), T_2(U)) \text{ be such that}

\[
\text{An upper bound on the payoff of task 2’ in a single task project is given by the payoff when } \hat{p}_2 = 1 \text{ and converges to zero as } \bar{p}_2 \text{ converges to zero.}
\]
Define
\[ T^{2**} = \max\{\min\{\bar{T}^2, \bar{T}^2(\bar{U})\}, T^{2*}\} \].

In the appendix we provide a condition on primitives that guarantees that the parameters fall in the cheap first-task incentives case (Condition 1). Formally, this condition holds if and only if the agents have strict incentives to exert effort in the first task when rewarded with the principal’s optimal single-task experimentation thresholds in the second task. Condition 1 holds, in particular, if \( \bar{U} \) is sufficiently large so that \( T^{2**} = \bar{T}^2 \), and \( \kappa^2 \) is sufficiently large relative to \( \kappa^1 \) or \( r \) is sufficiently small.\(^{41}\) When parameters fall in this region the cost of the first-task experimentation is small relative to the second task’s, as the former’s instantaneous cost is small and the latter’s length is long.

**Proposition 7** (Cheap first-task incentives). The parameters fall in the cheap first-task incentives case if and only if Condition 1 holds. Agents work at maximum effort until threshold \( \bar{T}_1 \) in the first task and in the second task they exert maximum effort until time threshold \( T^{2**} \). \( T^{2**} \) is increasing in the outside option \( \bar{U} \). The time zero lump-sum transfer is constant in \( i \) and \( W_{i,0} > 0 \) implies that the experimentation thresholds are efficient in both tasks after every history.

In the cheap incentives case the agents obtain the same expected payoff regardless of who succeeds. Unlike in the costly first-task incentives case, the principal does not gain from replacing agents who do not succeed in the first task.

### 4.2.3 Intermediate incentive costs in the first task

The following proposition describes the optimal contract when the first task has intermediate costs. The main difference with the other two cases is that there are first-task histories after which the agents are rewarded via a fine-tuning of the second-task experimentation assignments so that they are just indifferent between exerting effort in one instant or the next. At other histories, agents either receive bonuses as in the contract described in Proposition 6 or do not receive bonuses and their allocation is undistorted as in Proposition 7.

This case obtains when neither the cheap nor the costly incentives cases hold.

**Proposition 8** (Intermediate cost). In the intermediate costs case there are time thresholds \( t^1, t^2 \geq 0 \) such that for \( t \in [t^1, t^2] \), \( u_{i,t} \) satisfies the differential equation (9) and \( u_{i,t} = v_{i,t}(T_i(i, t)) \).

Proposition 8 says that there must be histories in which the contract does not reward agents with bonuses but with assignments of experimentation in the second task.

\(^{41}\)These conditions guarantee that 1 in Condition 1 holds. Large \( \kappa^2 \) guarantees \( \Pi^2 < v \) and either small or \( r \) or small \( \kappa^1 \) will imply that 1 holds.
The contract in the intermediate costs case cannot be derived in closed form. The experimentation thresholds of successful and unsuccessful agents must satisfy a joint optimality condition: the payoff function $u_{i,t}$ must make the agent just indifferent between exerting effort in the present instant and the consecutive instant and, at the same time, coincide with $i$’s expected payoff in the second task, which depends on $i$’s experimentation threshold.\footnote{The conditions that must be satisfied in the intermediate costs case are in appendix section B.4.}

As in the previous cases, agents with a higher outside option receive better experimentation allocations, at least in expectation, instead of lump-sum transfers.

### 4.3 Optimal disclosure of breakthroughs

We saw that in the one-task case delaying disclosure of players’ breakthroughs is unprofitable as it does not translate into a reduction in costs. In the two-task model there is an additional reason for which the principal may wish to delay effort: it reduces the costs from the public good rents. However, it also delays the start of the unsuccessful player’s second-task experimentation. The following Proposition shows that the latter effect dominates in the costly and cheap incentives case.

**Proposition 9.** Suppose that the parameters of the problem fall in the costly or cheap incentives case and, breakthroughs are verifiable. Then the principal discloses breakthroughs to both agents as soon as they occur.

To gain some intuition for this result note first from the argument for the one-task case it is not profitable to delay the disclosure of a breakthrough in the second task.\footnote{A delay in disclosure in the second task can potentially improve incentives in the first task by increasing the reward for success. However, because the second breakthroughs have no value for the principal and the agent incurs a cost for effort, any such increased reward can be achieved more cost effectively by granting a transfer.} Second, the agent’s reward for success can be separated into two terms as in equation (11). The first term corresponds to the reward due to procrastination rents and the second term corresponds to the payoff from the distribution of the disclosure of the other agent’s successes from an ex-ante perspective. As before, delaying disclosure does not reduce procrastination rents. It may, however, reduce the public-good rents of the successful agent. This reduction in wages comes at a cost to the principal, because delaying disclosure delays the unsuccessful agent’s experimentation in the second task. Proposition 9 shows that delaying disclosures is unprofitable, as it is unprofitable to delay second-task efforts in the model with observable breakthroughs.
5 Extension: more general learning

We now analyze a more general setting in which agents learn about the project they are involved in as they work and characterize under what conditions the optimal contract must provide procrastination rents to the agents. Since the presence of procrastination rents is the key feature of our model, this exercise gives us an idea of which settings will yield findings similar to the ones we described. In this more general setting, the agents’ work affects the rate at which a verifiable signal—say, for instance, a breakthrough or a breakdown—arrives.

Suppose there are \( K \) signals \( \{s^i_1, \ldots, s^i_K\} \) that can be produced by each player \( i \). Signal \( k \) is produced by \( i \) at time \( t \) at instantaneous rate \( \lambda^i_{k,t} a_{i,t} \), where agent \( i \)'s effort is in interval \([0, \bar{a}_i]\) and has a cost of effort that is linear in \( a_{i,t} \), given by \( \kappa_i a_{i,t} \). The rate \( \lambda^i_{k,t} \) is governed by a differential equation that depends on the joint effort of all agents:

\[
\dot{\lambda}^i_{k,t} = f^i_{ik}(\sum_j a_{j,t}, \lambda^i_{k,t}),
\]

for \( k \in \{1, \ldots, K\} \) with \( f^i_{ik} \) continuously differentiable and with \( \lambda^i_{k,0} \) at time zero given by \( \lambda^i_{k,0} \).\(^{44}\)

Let \( \lambda^i_{k,t}(\bar{a}_i, \tau \leq t, (a_{-i,\tau})_{\tau \leq t}) \) denote the rate \( \lambda^i_{k,t} \) at time \( t \) when agent \( i \) chooses effort function \( \bar{a}_i, \tau \) and agents other than \( i \) choose joint effort function \( a_{-i,\tau} \) for each \( \tau \leq t \).

Proposition 10 below shows that as long as there is a signal \( k \) such that \( f^i_{ik} \) is non-decreasing in \( a_{i,t} \)—that is, more effort increases the rate at which signal \( k \) arrives—the principal can provide incentives for agent \( i \) to exert the maximum effort, \( a_{i,t} = \bar{a}_i \), while extracting full surplus. The principal does not have to provide information rents, and can pay a bonus that exactly compensates each agent for the cost of effort in expectation whenever the first signal to arrive is signal \( k \). Since the arrival rate of the signal in non-decreasing in effort, \( i \) has no incentive to procrastinate: by delaying effort the agent makes the arrival of a reward in the future less likely and sees his expected payoff diminished. In the context of our model, this includes the case in which the tasks are known to be feasible with probability 1.

Let \( w^i_{s^j_k,t} \) denote the bonus payment to agent \( i \) when agent \( j \) produces signal \( s^j_k \). The payoff of agent \( i \) is given by

\[
\int_0^t \left( \sum_{k,j} \lambda^j_{k,t} w^j_{s^j_k,t} a_{j,t} \right) e^{-\sum_{k,j} \int_0^t \lambda^j_{k,s} a_{j,s} ds - rt} dt.
\]

\( a_{i,t} \lambda^i_{k,t} \) is the probability that agent \( i \) receives signal \( s_k \) at time \( t \) and \( e^{-\sum_{k,j} \int_0^t \lambda^j_{k,s} a_{j,s} ds} \) is the probability that no signal has arrived until time \( t \).

\(^{44}\)In the model in the main body of the paper the only signals are breakthroughs and their arrival rate is \( p_t \). The function \( f^i \) does not depend on \( i \) and is given by \( f^i(x,y) = -xy \).
Proposition 10 (No information rents). Suppose there is $k$ such that $f_k^i$ is non-decreasing in $a_{i,t}$. The principal can give incentives for $a_{i,t} = \bar{a}_i$ for every $t$ by giving bonus

$$w_{i,t}^{d_i} = \frac{\kappa_i}{\lambda_{k,t}^i ((\bar{a}_i, \tau)_{\tau \leq t}, (a_{i-1}, \tau)_{\tau \leq t})},$$

when signal $k$ is realized and setting $w_{i,t}^{d_i} = 0$ if $l \neq k$ or $j \neq i$.

Proof. If an agent deviates from $a_{i,t} = \bar{a}_i$ and exerts lower effort in a positive measure set then $\lambda_{k,t}^i < \lambda_{k,t}^i ((\bar{a}_i, \tau)_{\tau \leq t}, (a_{i-1}, \tau)_{\tau \leq t})$, thereafter and the expected payoff from positive effort is weakly negative. Thus, once an agent deviates to effort below $\bar{a}_i$ he exerts zero effort forever after. The payoff of the agent who deviates is zero which coincides with the payoff from exerting maximum effort at all times.

In the setting of the main body of the paper, if $p_0 = 1$ then $p_t = 0$ and the hypothesis of Proposition 10 holds. Thus, the principal can trivially induce effort in an arm that continues to produce profits once uncertainty is resolved.

A converse of the previous result also holds. Suppose there is a time interval $[t', t'']$ such that if each agent $j$ has exerted maximum effort $\bar{a}_j$, $f_k^i(\sum_j a_{j,t}, \lambda_{k,t}^i)$ is decreasing in $a_{i,t}$ for $t \in [t', t'']$ for all $k$. Then, the optimal contract must provide information rents. This result can be explained using the dynamic programming heuristic. Let $V_{i,t}$ denote the expected payoff of agent $i$ at time $t$. $V_{i,t}$ must satisfy

$$V_{i,t} = \left( \sum_k \lambda_{k,t}^i w_{i,t}^{d_i} - \kappa_i \right) a_{i,t} dt + \left( 1 - \left( \sum_k \lambda_{k,t}^i a_{j,t} \right) dt \right) V_{i,t+dt} + o(dt).$$

If the principal is extracting full surplus, $\left( \sum_k \lambda_{k,t}^i w_{i,t}^{d_i} - \kappa_i \right) = 0$ and $V_{i,t} = 0$ at every $t$. By exerting zero effort at time $t$, the agent can obtain a strictly positive surplus, as by slacking in interval $[t', t'']$ the agent obtains $V_{i,t} > 0$ (see section 2.2 for a similar argument).

Proposition 11 (General learning). Suppose there is a time interval $[t', t'']$ and signal $k$ such that if each agent $j$ has exerted maximum effort $\bar{a}_j$ up to time $t'$, $f_k^i(\sum_j a_{j,t}, \lambda_{k,t}^i)$ is decreasing in $a_{i,t}$ for $t \in [t', t'']$. Then, agent $i$ receives a strictly positive expected payoff.

From Proposition 11 many conclusions of the model apply to more general settings in which the rates at which agents produce successes or, more generally, produce any signal, varies as a function of how much effort they have put in. An agent receives rents as long as the learning process is such that the rate at which verifiable signals arrive has slope decreasing in his effort in some time interval. Thus, any learning process in which, at any point during the project, the agent
becomes more pessimistic about obtaining any verifiable signal (such as a success or a failure) must provide information rents. This includes settings in which learning is not about the quality of the project. As an example, suppose workers become tired as the week goes by, their tiredness causes them to produce successes at a lower rate and this tiredness is increasing in the amount of private effort that they’ve exerted so far. In such a setting a principal must provide information rents to induce effort as well. 45

6 Conclusions

This paper analyzes how to optimally design a contract that gives incentives to innovate to one or two agents. We show that incentives can be provided by simple history contingent bonus contracts. Agents receive information rents to prevent them from delaying effort over time. These rents are increasing with the amount of leeway to experiment that the agents are given. As a result, and in order to reduce information rents, the principal has the agents stop experimentation early compared to the first best.

As projects require multiple successful experiments, the contracts have two novel characteristics. First, the agents receive rents to prevent them from free-riding on other agents’ discoveries in early periods. Second, rewards and punishments are implemented by experimentation assignments. As a result, the optimal contracts for symmetric agents exhibit asymmetries that grow over time. To reduce the public-good rents, the principal may exclude some agents from working, even in the absence of another profitable project. Agents’ contracts are sensitive to early successes. Agents who succeed see their experimentation assignments increased, receiving bigger bonuses when they succeed and having more opportunities to do so.

Our paper has the following empirical implications. First, within a firm workers who obtain successes or promotions are likely to be credited with future successes in the same project as long as the project remains risky even if the workers are equally productive. This observation may be related to the empirical finding that workers who are promoted or receive raises are likely to receive another promotion in rapid succession. Our model predicts that workers who have a higher outside option will exhibit more serial correlation in raises and promotions.46,47 Second, consider

45If the costs of effort are non-linear, it is plausible that the agent must be prevented from front-loading effort. In such case, agents may have to be given rents even as they become more optimistic about the state of the project.
46Baker et al. (1994) find serial correlation in raises and promotions in their study of personnel data of a firm. As an explanation they propose persistent unobserved heterogeneity which drives fast advancement of some workers.
47To make this point in our model we would need at least three tasks. Using the intuitions of the two period model, in a three period model a worker who succeeds would be given a longer experimentation threshold in the second task which would make him more likely to succeed and receive a longer threshold in the third task. The techniques in this paper would be useful to solve the three task model. We do not solve it because we have not proven that the three period model is concave. We believe it has to be shown numerically.
a project that requires substantial investments. If the first stage of a project is relatively safe, that is, its prior probability of being attainable is high, we expect to see fewer workers participating in the early stages. In this case, the rents to free-riding are high relative to the benefit of competition in early stages. In our model the principal is able to observe each agent’s results. If the principal is less able to observe individual performance or there are other competing projects to which agents can be allocated, we should expect even fewer agents participating in the initial stages. Third, in expectation bonuses can be higher in environments with more uncertainty than in environments in which the task is of known quality. Agents receive zero rents in safe projects. Fourth, successful workers should be rewarded with promotions earlier in the project and bonuses later on. Workers who do not succeed are assigned less responsibility in the same project–compared to the successful ones—or are assigned to tasks that give them less information rents. Tasks that give less information rents can be tasks that carry less risk, are easier to perform (less costly), or have a slow arrival rate. Finally, the model also provides a rationale for why jobs comprised of different tasks provide different wages. One could test whether jobs in which workers are able to learn more provide higher wages.48

References


48In the model, because agents are risk neutral the agents only receive bonuses at certain times and do not receive monthly wages. If the agents were risk averse the principal would have to provide insurance in the form of smoother wages.


A Appendix: Model and benchmark

A.1 Bonus contracts are without loss

To see that it is without loss to restrict attention to bonus contracts, suppose the principal could offer each agent \( i \) a wage schedule \( \tilde{w}_i \) contingent on the public history. The wage schedule at time \( t \) consists of a flow payoff \( \tilde{w}^f_{i,t} \in \mathbb{R} \) and lump-sum transfers \( \tilde{w}^l_{i,t} \in \mathbb{R} \). That is, heuristically the revenue accruing to the agent over the time interval \( [t, t + dt] \) is

\[
\tilde{w}^f_{i,t} dt + \tilde{w}^l_{i,t}.
\]

The wage schedule \((\tilde{w}^f_{i,t}, \tilde{w}^l_{i,t})\) is adapted to the \( \sigma \)-algebra induced by the public histories in set \( \mathcal{H}^t \) and maps public histories to \( \mathbb{R} \).

Let \( \tilde{w}_i : \mathcal{H}^t \to \mathbb{R} \) denote a payment scheme as a function of history for \( i \in \{1, \ldots, n\} \). Let’s see that there is a bonus contract of the form \( w = (w_i(h'), W_i,0)_{i,h' \in \mathcal{H}^t} \), where \( w_i(h') \) denotes the amount that agent \( i \) gets paid at history \( h' \) and \( W_i,0 \) denotes the transfer at time zero that gives the same payoff to principal and agent after each history.

In what follows we assume the project has \( N \geq 2 \) uncertain tasks.

Let \( h \in \mathcal{H}^t \) be a terminal history in which breakthroughs arrive at times \( \tau_1, \ldots, \tau_J \) to agents \( k_1, \ldots, k_J \) for \( 0 \leq J \leq N \). If \( J = 0 \), \( h \) is the history in which no breakthroughs are attained. Let \( h^\tau_j \) denote the history \( h \) truncated to time \( \tau_j \), time \( \tau_j \) inclusive. Define \( \tau_0 = 0 \) and let \( \tilde{w}_i^j(0, h^{\tau_j-1}) \) denote the discounted payoff that contract \( \tilde{w}_i \) gives to agent \( i \) at the history in which the game ends with no verifiable signals at task \( j \) after history \( h^{\tau_j-1} \).

Define:

\[
W_{i,0} = \tilde{w}_i^1(0, h^0),
\]

and

\[
w_{i,\tau_j}(h^{\tau_j}) = \left( \tilde{w}_i^{j+1}(0, h^{\tau_j}) - \tilde{w}_i^j(0, h^{\tau_j-1}) \right) e^{\tau_j}.
\]

Contract \( w = (w_{i,t}, W_{i,0}) \) is a bonus contract that gives the same expected payoff after each history to all players as contract \( \tilde{w}_i \).

To see that bonus contracts are without loss even when contracts must satisfy limited liability note that if the original contract satisfies limited liability, that is \( \tilde{w}^f_{i,t}, \tilde{w}^l_{i,t} \geq 0 \) at every history, then the associated bonus contract satisfies \( w_{i,\tau_j}, W_{i,0} \geq 0 \).

\[49\]The limited liability condition can be replaced by the weaker condition

\[
\int_0^\infty e^{-r_h} \tilde{w}^f_{i,t}(h) ds + \sum_{k \in (h)} \tilde{w}^l_{i,t_k}(h) e^{-r_k} \geq 0,
\]

at every history \( h \). The reason is that the only contracts that (LL) rules out relative to (14) are those in which the
A.2 Derivations for section 3.1

Solving the differential equation given by equation (2) in Proposition 1 we obtain

\[
 w_{i,t} = \kappa_i \left( \exp \left( - \int_t^T (r - a_{i,s}) \, ds + \int_t^T a_s \, ds + x_0 \right) + 1 \right) + e^{rt} \int_t^T \kappa_i e^{-r\tau - \int_0^\tau a_{i,s} \, ds + x_0} \, d\tau.
\]

The agent’s payoff is given by

\[
 \int_0^T (p_i w_{i,t} - \kappa_i) a_{i,t} e^{-\int_0^t (p_{i,s} a_s + r) \, ds} \, dt.
\]

Replacing the expression for \( w_{i,t} \), i’s expected payoff from time \( \hat{t} \) on is given by

\[
 V_{i,\hat{t}} = (1 - \bar{p}) \int_{\hat{t}}^T a_{i,t} \left( \int_{\hat{t}}^T \kappa_i e^{\int_{\hat{t}}^\tau a_{s,t} \, ds - r\tau} \, d\tau + \kappa_i e^{-rt \left( e^{-\int_{\hat{t}}^T (r - a_{i,s}) \, ds} - 1 \right)} \right) \, dt. \tag{15}
\]

Note that this payoff only depends on \( i \)'s effort function.

To gain intuition for the shape of the optimal bonus contract, note that approximating \( (1 - e^{-a_{i,t} p_i dt}) \) and \( e^{-(r + p_t (a_{i,t} + a_{-i,t})) dt} \) by their second order Taylor expansion we obtain that equation (3) in footnote 20 can be approximated by

\[
 V_{i,t} = \left( w_{i,t} \left( dt p_{i,t} - \frac{1}{2} a_{i,t} (dt p_{i,t})^2 \right) - \kappa_i dt \right) a_{i,t} + \left( \frac{1}{2} (dt p_{i,t} (a_{i,t} + a_{-i,t}) + r) - (dt (a_{i,t} + a_{-i,t}) p_{i,t} + r) + 1 \right) V_{i,t+dt}
\]

Replacing \( V_{i,t+dt} \) in a similar manner and approximating \( p_{t+dt} = p_t - p_t (1 - p_t) (a_{i,t} + a_{-i,t}) dt \) we obtain the following expression for \( V_{i,t} \)

\[
 V_{i,t} = dt (p a_{i,t} w_{i,t} - \kappa_i a_{i,t}) - \frac{1}{2} dt^2 w_{i,t} (p a_{i,t})^2 +

\left( (1 - (r + p (a_{i,t} + a_{-i,t})) dt + (r + p(a_{i,t} + a_{-i,t}))^2 dt^2 / 2) \times \right.

\left( dt (p a_{i,t+dt} w_{i,t+dt} - \kappa_i a_{i,t+dt}) - dt^2 p a_{i,t+dt} w_{i,t} \left( (1 - p)(a_{i,t} + a_{-i,t}) + \frac{p a_{i,t+dt}}{2} \right) + \right.

\left. (1 - dt (p(a_{i,t+dt} + a_{-i,t+dt}) + r) + \frac{1}{2} (p(a_{i,t+dt} + a_{-i,t+dt}) + r)^2 +

+ p(1 - p)(a_{i,t} + a_{-i,t})(a_{i,t+dt} + a_{-i,t+dt}) dt^2) V_{i,t+2dt} \right) + o(dt^2), \tag{16}
\]

where we have dropped the time subscripts on \( p \) for simplicity.

From equation (15) we can see that \( V_{i,t+2dt} \) does not depend on \( \varepsilon \) as the total experimentation payoff of achieving a breakthrough is less than the payoff of not achieving it. These contracts are not optimal for the principal. By reducing the transfer after the history in which there is no success the agents’ incentives are improved and the principal reduces her expenses.

42
from time \( t + 2 dt \) on does not depend on \( \varepsilon \). Thus, the effect of delaying an amount \( \varepsilon \) of effort from time interval \([t, t + dt]\) to interval \([t + dt, t + 2 dt]\), \( \frac{\partial V_{i,t}/\partial \varepsilon}{\partial (dt)^2} \), can be approximated by

\[
- \frac{\partial V_{i,t}}{\partial a_{i,t}} + \frac{\partial V_{i,t}}{\partial a_{i,t+dt}} = dt^2 (-p_t (a_{-i,t} + r) (w_{i,t} - \kappa_t) + \kappa_t (-p_t) r + \kappa t + dt (w_{i,t+1} - w_{i,t}) + o(dt^2).
\]

(17)

Dividing by \( dt^2 \) and taking the limit \( dt \to 0 \) yields

\[
\dot{w}_i = (a_{-i} + r)(w_i - \kappa_t) - r \kappa_t \varepsilon^x.
\]

Thus, the contract offered by the principal makes agent \( i \) just indifferent between exerting effort in the present instant and shifting it to the next.

To derive equation (6), note that replacing \( w_{i,t} = \pi \) in equation (17) yields

\[
\frac{\partial \Pi_{i,t}/\partial \varepsilon}{\partial (dt)^2} = (-p_t (a_{-i,t} + r) (\pi - \kappa) + \kappa_t (-p_t) r + \kappa r).
\]

Writing \( V_{j,t} \) as in (15) yields \( -\frac{\partial V_{i,t}}{\partial a_{i,t}} + \frac{\partial V_{i,t}}{\partial a_{i,t+dt}} = p_t a_{j,i} (w_{j,t} - \kappa) \). Thus, replacing \( w_{j,t} = \pi \) we obtain

\[
\frac{\partial \Pi_{j,t}/\partial \varepsilon}{\partial (dt)^2} = p_t a_{j,i} (\pi - \kappa) \text{ and } \sum_k^n \frac{\partial \Pi_{k,j}/\partial \varepsilon}{\partial (dt)^2} = -(p_t \pi - \kappa)r.
\]

### A.3 Proof of proposition 1

The principal chooses \( w_{i,t} : \mathcal{H}_t \to \mathbb{R}_+ \) and \( a_i : \mathcal{H}_t \to [0, 1] \) measurable with respect to history to maximize her profits. The principal’s objective is to find

\[
\max_{a_{i,t}, w_{i,t}} \sum_i r \int_0^\infty p_t a_{i,t} (\pi - w_{i,t}) e^{-\int_0^t (p_t a_{i,t} + \kappa_t) ds} dt,
\]

such that \( a_i : \mathbb{R}_+ \to [0, 1] \) maximizes

\[
r \int_0^\infty (p_t w_{i,t} - \kappa_t) a_{i,t} e^{-\int_0^t (p_t a_{i,t} + \kappa_t) ds} dt.
\]

The belief evolves according to

\[
\dot{p}_t = -p_t (1 - p_t) (a_{i,t} + a_{-i,t}),
\]

where \( a_{-i,t} = \sum_{j \neq i} a_{j,t}, a_s = \sum_i a_{i,s} \).

The characterization of the optimal bonus contract \( w_{i,t} \) offered to each agent \( i \) is done in the following steps. We first derive necessary conditions that \( w_{i,t} \) must satisfy to implement a given effort
schedule \( a_{i,s} \). We then find the bonus contract that minimizes the principal’s cost among the class of bonus contracts that satisfy the necessary conditions. Finally, we show that the cost-minimizing bonus contract satisfies sufficient conditions for optimality and is, therefore, the optimal bonus schedule for a given effort \( a_{i,s} \).

The agent’s problem

We now write the agent’s problem, given a bonus contract \( w_{i,t} \), and derive necessary conditions for the agent’s choice of effort given the contract using Pontryagin’s maximum principle. Let \( T_i = \sup \{ \tau | a_{i,\tau} > 0 \} \). \( T_i \) denotes the latest time at which agent \( i \) exerts effort. We will see that at the optimal effort functions \( T_i \) is finite for each \( i \). Suppose the principal wants to implement effort function \( a_{i,s} \) for each agent \( i \). The belief, \( p_t \), about the feasibility of the task evolves according to

\[
\dot{p}_t = \bar{p}e^{-\int_0^t a_s \, ds} - \kappa \bar{p}e^{-\int_0^t a_s \, ds} - \kappa (1 - \bar{p})e^{-\int_0^t a_s \, ds}.
\]

Thus, agent \( i \)'s problem can be written as

\[
\max_{a_i} \int_0^{T_i} \left( w_{i,t} \bar{p}e^{-\int_0^s a_s \, ds} - \kappa \bar{p}e^{-\int_0^s a_s \, ds} + (1 - \bar{p}) \right) a_{i,t} e^{-rt} \, dt,
\]

subject to

\[
\dot{y}_t = a_i + a_{-i}.
\]

In the agent’s optimal control problem \( y_t \) is a state variable and \( a_{i,t} \) is a control variable. The Hamiltonian is

\[
H(a_{i,t}, y_t, \gamma_t) = (\bar{p}w_{i,t}e^{-y_t} - \kappa \bar{p}e^{-y_t} - \kappa (1 - \bar{p})) a_{i,t} e^{-rt} + \eta_{i,t}(a_{i,t} + a_{-i,t}),
\]

where \( \eta_{i,t} \) is the costate variable associated to \( y \). From Theorem 22.26 in page 465 of Clarke (2013), for any measurable \( w_{i,t} \), \( \eta_{i,t} \) is an absolutely continuous function that evolves according to

\[
\dot{\eta}_{i,t} = -\frac{\partial H(a_{i,t}, y_t, \gamma_t)}{\partial y} = \bar{p}(w_{i,t} - \kappa) e^{-y_t} a_{i,t} e^{-rt}.
\]
Moreover, \( a_{i,t} \) maximizes \( H(a_{i,t}, y_t, \gamma_{i,t}) \) and therefore maximizes
\[
\left( (\bar{p} w_{i,t} e^{-\gamma_{i,t}} - \kappa \bar{p} e^{-\gamma_{i,t}} - \kappa(1 - \bar{p})) e^{-rt} + \eta_{i,t} \right) a_{i,t}. \tag{21}
\]
Define
\[
\gamma_{i,t} \equiv \left( (\bar{p} w_{i,t} e^{-\gamma_{i,t}} - \kappa \bar{p} e^{-\gamma_{i,t}} - \kappa(1 - \bar{p})) + \eta_{i,t} e^{rt} \right) \frac{1}{1 - \bar{p}}. \tag{22}
\]
Since \( a_{i,t} \) maximizes (21) we have \( \gamma_{i,t} > 0 \implies a_{i,t} = \bar{a}_i \) and \( \gamma_{i,t} < 0 \implies a_{i,t} = 0 \).

The transversality condition, as \( y \) is unrestricted at \( T_i \), is \( \eta_{i,T_i} = 0 \) which implies
\[
\gamma_{i,T_i} = \left( -\kappa - e^{-x_T} \kappa + e^{-x_T} \bar{w}_{i,T_i} \right), \tag{23}
\]
where \( x_t = \int_0^t y_s ds + \log \left( \frac{1 - \bar{p}}{p} \right) \). Conditions (20) and (23) are necessary for the agent’s choice of effort.

Given \( \eta_{i,t} \), from equation (22) we see that if \( \gamma_{i,t} > 0 \) for \( t \) in some interval \([\bar{t}_1, \bar{t}_2]\) then the principal may lower \( w_{i,t} \), so as to decrease the expected payments to \( i \), without affecting the effort \( a_{i,t} \) that maximizes (21). Thus, of all wage schedules that satisfy agent \( i \)'s necessary conditions for effort function \( a_{i,s} \), the principal’s preferred one is such that \( \gamma_{i,t} = 0 \) or, equivalently, such that \( \eta_{i,t} \) satisfies
\[
\eta_{i,t} = -\left( \bar{p} w_{i,t} e^{-\gamma_{i,t}} - \kappa \bar{p} e^{-\gamma_{i,t}} - \kappa(1 - \bar{p}) \right) e^{-rt}. \tag{24}
\]
We will see that, for a given effort function \( a_{i,t} \), at the contract such that \( \gamma_{i,t} = 0 \) the necessary conditions above are also sufficient. Thus, the agent’s choice of effort under that contract is indeed \( a_{i,t} \). By the previous discussion, the contract obtained by setting \( \gamma_{i,t} = 0 \) is the one that minimizes the principal’s wage costs among contracts that satisfy the necessary condition (20).

Setting \( \gamma_{i,t} = 0 \) and replacing the expression for \( \eta_{i,t} \) in equation (24) into equation (20), we obtain
\[
0 = r\kappa + e^{-x_T} (r(\kappa - w_{i,t}) + (\kappa - w_{i,t})a_{i,t} + w_{i,t}). \tag{25}
\]

Fixing the value of \( w_{i,T_i} \), there is a unique bonus contract that satisfies the previous differential equation. In fact, integrating equation (25) we obtain
\[
\begin{align*}
\bar{w}_{i,t} &= \kappa \left( 1 - e^{-\int_t^{T_i}(r + a_{i,t})ds} \right) + e^{\int_t^{T_i}a_{i,t}ds+rt} \int_t^{T_i} e^{-rt} \int_t^{T_i} e^{\int_t^{s}a_{i,t}ds+dx_0} r\kappa dl \\
&+ e^{-\int_t^{T_i}(r + a_{i,t})ds} w_{i,T_i}.
\end{align*} \tag{26}
\]

If a solution to the agent’s problem exists conditions (20), (21) and (23) are necessary conditions for the agent’s problem. We next show that a solution to the agent’s problem exists and that these necessary conditions are also sufficient at the contract obtained by setting \( \gamma_{i,t} = 0 \). For
this purpose we first show that the problem has a solution and then we show that the necessary conditions are also sufficient.

The solution to the agent’s problem exists

Let’s see that agent \( i \)’s problem has a solution for each \( w_{i,t} \). By Theorem 23.11 in page 481 of Clarke (2013) agent \( i \)’s problem has a solution. In fact, the bonus wage, \( w_{i,t} \) is Lebesgue measurable in \( t \) and we have that

\[
\Lambda(t,x,a) = (\bar{p}w_{i,t}e^{-\gamma_I} - \kappa \bar{p}e^{-\gamma_I} - \kappa(1 - \bar{p})) a_{i,t}e^{-r}
\]

is Lebesgue measurable, convex in \( a_i \) and lower semicontinuous in \((y,a_i)\). The set of controls is bounded, the process \( \dot{y} = a_{-i} \) and \( a_i = 0 \) is admissible and makes the agent’s objective finite.

The necessary conditions are also sufficient

Let \( w_{i,t} \) be a bonus contract that satisfies equation (25) for \( t \leq T_i \) and is equal to zero for \( t > T_i \). We now show that an effort schedule \((a_{i,s})_{s \in \mathbb{R}} \neq 0 \) satisfies the agent’s IC when the principal offers bonus contract \( w_{i,t} \). Since the problem’s solution exists, if the effort function \( a_{i,s} \) does not satisfy IC there must be another function \( \tilde{a}_{i,s} \) and costate variable \( \tilde{\eta}_{i,s} \) that satisfy the necessary condition (20) and improves the agent’s payoff. Let’s see that such effort function and associated costate variable do not exist.

Replacing \( \gamma_{i,t} \) into equation (20) we obtain that the necessary condition for effort \( a_{i,t} \) is that there is an arc \( \gamma_{i,t} \) such that:

\[
\dot{\gamma}_{i,t} = \gamma_{i,t} r + r\kappa + e^{-x_i}(r(\kappa - w_{i,t}) + (\kappa - w_{i,t})a_{-i,t} + \tilde{w}_{i,t}).
\]

Let’s see that only \( \gamma_{i,t} = 0 \) for all \( t \) and effort function \( a_{i,t} \) (up to Lebesgue measure zero sets in \( t \) ) can satisfy \( i \)’s necessary condition (27) if the principal offers bonus contract \( w_{i,t} \). By contradiction, suppose there is a multiplier \( \hat{\gamma}_{i,t} \) and an effort function \( (\tilde{a}_{i,t})_t \neq (a_{i,t}) \) that satisfy the necessary condition (27) where we replace \( a_{i,t} \) by \( \tilde{a}_{i,t} \) and \( \gamma_{i,t} \) by \( \gamma_{i,t} \). Define \( \hat{x}_t = x_0 + \int_0^t \tilde{a}_{i,s} ds \) and let \( \tau_0 = \inf\{t|\hat{\gamma}_{i,t} \neq 0\} \).

Suppose that \( \tau_0 = 0 \). Since \( \hat{\gamma}_{i,t} \) is continuous there exists \( \varepsilon > 0 \) such that either \( \hat{\gamma}_{i,t} > 0 \) for \( t \in (0,\varepsilon) \) or \( \hat{\gamma}_{i,t} < 0 \) for \( t \in (0,\varepsilon) \). Consider the case in which \( \hat{\gamma}_{i,t} > 0 \) for \( t \in (0,\varepsilon) \). From condition (21) we have \( \tilde{a}_{i,t} = a_i \) for \( t \in (0,\varepsilon) \). From equation (27), because \( \hat{x}_t \geq x_t \), \( r\kappa + e^{-x_i}(r(\kappa - w_{i,t}) + (\kappa - w_{i,t})a_{-i,t} + \tilde{w}_{i,t}) \geq 0 \) implies \( r\kappa + e^{-x_i}(r(\kappa - w_{i,t}) + (\kappa - w_{i,t})a_{-i,t} + \tilde{w}_{i,t}) \geq 0 \). Thus, we must have \( \hat{\gamma}_{i,t} > 0 \), and, thus, \( \hat{\gamma}_{i,t} > 0 \) and \( \tilde{a}_{i,t} = a_i \) for every \( t \leq T_i \). However,

\[
\hat{\gamma}_{i,T_i} = (-\kappa + e^{-x_i}(w_{i,T_i} - \kappa)) \leq (-\kappa - e^{-x_i}r\kappa + e^{-x_i}w_{i,T_i}) = 0,
\]
where the first equality comes from the transversality condition for \((\tilde{y}_{l,t}, \tilde{a}_{l,t})\), and the inequality from \(x_T = x_0 + \sum_i \int_0^{T_l} a_{l,s} ds \leq x_0 + \sum_i \int_0^{T_l} \tilde{a}_{l,s} ds = \tilde{x}_T\) and \(w_{l,T_l} \geq \kappa\). This contradicts \(\tilde{y}_{l,T_l} > 0\) by an analogous argument, assuming \(\tau_0 = 0\) and \(\tilde{y}_{l,T_l} < 0\) for \(t \in (0, \epsilon)\) also leads to a contradiction.

Now, suppose \(\tau_0 > 0\) and let \(t' = \inf \{t \mid x_t > \tilde{x}_t\} \) and \(t'_+ = \inf \{t \mid x_t < \tilde{x}_t\} \). Suppose \(t' < t'_+\). Since \(r\kappa + e^{-\kappa t}(r(\kappa - w_{l,t}) + (\kappa - w_{l,t}) a_{l,t} + \tilde{w}_{l,t}) = 0\) for \(t \in (0, t'_-)\), then \(r\kappa + e^{-\kappa t}(r(\kappa - w_{l,t}) + (\kappa - w_{l,t}) a_{l,t} + \tilde{w}_{l,t}) < 0\) for \(t \in (t', t'_+ + \epsilon)\) which implies \(\tilde{y}_{l,T_l} < 0\) for \(t \geq t'\) and \(\tilde{a}_{l,t} = 0\) for \(t \geq t'\). However, \(\tilde{y}_{l,T_l} = (-r\kappa + e^{-\kappa t} e^{s\kappa}w_{l,T_l}) \geq (-r\kappa + e^{-\kappa t} e^{s\kappa}w_{l,T_l}) = 0\) since \(\tilde{x}_{T_l} = x_0 + \sum_i \int_0^{T_l} \tilde{a}_{l,s} ds \leq x_0 + \sum_i \int_0^{T_l} a_{l,s} ds = x_T\), which contradicts \(\tilde{y}_{l,T_l} > 0\). An analogous argument finds a contradiction in the case in which \(t'_+ > t'_+\).

### A.4 Proof of Proposition 2

To establish Proposition 2 we need to show that the principal’s choice of effort functions \(a_{i,t}\) for \(i \in \{1, \ldots, n\}\) is given \(a_{i,t} = \bar{a}\) for \(t \leq T^*\) and \(a_{i,t} = 0\) for \(t > T^*\).

#### Agents exert maximum effort

The effect of shifting effort from a later to an earlier time is two-fold. It increases surplus and also increases the agents’ wages. Let’s see that the first effect dominates the second one and the payoff of the principal that equals the surplus minus the agents’ payoff must increase when effort is shifted to earlier times. From equation (6) and the discussion that leads to equation (6), shifting agent \(i\)’s effort from interval \([t + dt, t + 2dt]\) to interval \([t, t + dt]\) has a positive second order effect on surplus from time \(t\), \(\tilde{N}_t\), and at most a third order effect on agent \(i\)’s payoff, \(V_{i,t}\). Now, the surplus from time zero to \(t\) is unaffected by the change. However, the shift of effort increases agent \(i\)’s payoff. Let’s see that the effect on each agent \(i\)’s expected payoff from time 0 to time \(t\) is at most of third order. Let \(a_{i,t}\) denote agent \(i\)’s effort function and let \(\bar{a}_{i,t}\) denote the effort after the shift in effort. From equation (15), the change in agent \(i\)’s payoff from 0 to \(t\) is proportional to

\[
\Delta_e = \int_0^t \int_0^{t+2dt} \kappa r \left( e^{h_1 \bar{a}_{i,s}} \alpha_{i,s} - e^{h_1 a_{i,s}} \alpha_{i,s} \right) \, d\tau \, d\tilde{t} = \int_t^{t+2dt} \int_0^t \kappa r \left( e^{h_1 \bar{a}_{i,s}} \alpha_{i,s} - e^{h_1 a_{i,s}} \alpha_{i,s} \right) \, d\tau \, d\tilde{t} = \kappa r \int_0^{t+2dt} e^{-\kappa t} \left( e^{b_0 \bar{a}_{i,s}} \alpha_{i,s} - 1 \right) \left( e^{b_0 a_{i,s}} \alpha_{i,s} - e^{b_0 a_{i,s}} \alpha_{i,s} \right) \, d\tau,
\]

where the first equality is obtained by changing the order of integration and the second by integrating over \(\tilde{t}\) and noting that \(\int_0^t a_{i,s} \, ds = \int_0^t \bar{a}_{i,s} \, ds\). The previous expression is less than or equal than

\[
\kappa \rho e^{-\kappa t} \left( e^{b_0 a_{i,s}} - 1 \right) \left( \int_t^{t+dt} e^{(\bar{a} + \epsilon)(\tau - t) - e^{\bar{a}(\tau - t)}} \, d\tau + \int_{t+dt}^{t+2dt} e^{(\bar{a} + \epsilon)(\tau - t) - e^{\bar{a}(\tau - t)}} \, d\tau \right).
\]
Ignoring the terms that don’t depend on ε, and constants, and integrating over τ yields

$$\frac{1}{\bar{a} + \varepsilon} e^{(\bar{a} + \varepsilon)dt} - 1 + \frac{1}{\bar{a} - \varepsilon} \left( e^{2\bar{a}dt} - e^{(\bar{a} + \varepsilon)dt} \right).$$

We are interested in the effect on Δε of increasing ε from zero when dt is small. Taking the derivative with respect to ε yields

$$\frac{dte^{(\bar{a} + \varepsilon)dt}}{(\bar{a} + \varepsilon)} - \frac{(e^{(\bar{a} + \varepsilon)dt} - 1)}{(\bar{a} + \varepsilon)^2} = e^{3\bar{a}(\bar{a} + \varepsilon)dt/2}. $$

Approximating the previous expression up to the second order using the approximation $e^x = 1 + x$ yields

$$\left(\frac{dt(1 + (\bar{a} + \varepsilon)dt)}{(\bar{a} + \varepsilon)} - \frac{dt((\bar{a} + \varepsilon)dt + 1)}{(\bar{a} - \varepsilon)} + \frac{(2\bar{a}dt - (\bar{a} + \varepsilon)dt)}{(\bar{a} - \varepsilon)^2}\right)_{\varepsilon = 0} = 0.$$

Thus, close to zero the effect of changing ε is at most of third order in dt on i’s utility—and, hence, in the principal’s utility—from time zero to time t. Since the principal obtains a second order gain from the increase in surplus as shown in equation (6), the principal is made better off by shifting i’s effort from $[t + dt, t + 2dt]$ to $[t, t + dt]$. This shows that at the optimal contract the agents must exert maximum effort until a deadline.

**Experimentation thresholds are symmetric** Let’s now derive the thresholds at which the agents stop experimenting. Suppose agent i stops last. The first order condition for the choice of $T_i$ is:

$$\hat{a} \int_0^{T_i} p_i a_i (\pi - w_{ij}) e^{-\int_0^t (p_i a_i + r) ds} dt = e^{-x_{T_i}} \left( \pi - e^{x_{T_i} + T_i \bar{a}} \kappa - \kappa \right) \bar{a} = 0. \quad (28)$$

The derivative is decreasing in $T_i$ and therefore there is a unique $T_i$ that solves (28). Let $\bar{T}$ be the set of agents who stop at time $T_i$ and let $\bar{n} = |\bar{T}| + 1$. Let j be the player who stops second to last. Since increasing $T_j$ affects the wages of j and also those of the agents in set $\bar{T}$, the first order condition with respect to $T_j$ is given by

$$e^{-rT_j + \bar{T}} \frac{e^{-\bar{a}(r + \bar{T}_j)} \left( r + \bar{a} \left( -2 + e^{(r + \bar{a}(1 + \bar{n})) (T_j - \bar{T})} + \bar{n} \right) \right)}{r + \bar{a}(1 + \bar{n})} e^{-x_{T_j} - T_j \bar{a}} = 0.$$

However, this expression cannot be zero when $T_j < T_i$. In fact, the term * is strictly greater than one which together with (28) implies that the the term in parenthesis is strictly positive. To see that * is greater than one note that at $T_i = T_j$ it is equal to one. The derivative of the numerator in * is
given by
\[ e^{\bar{a}(T_i-T_j)} \left( e^{(r+\bar{a}(-1+\bar{n}))(T_j-T_i)} (-r + \bar{a}) + (r + \bar{a}(-2+\bar{n})) \bar{n} \right), \]

which is positive whenever \( T_i > T_j \). This shows that the optimal contract for symmetric agents is symmetric.

We can now compute the optimal contract by solving its differential equation. Let \( T^* \) denote the threshold at which agents stop experimentation. By replacing the condition \( w_i(T^*) p_{T^*} = \kappa \) for each agent \( i \) into (26) (which is implied by the transversality condition, \( \gamma_i, T_i = 0 \)) we obtain

\[ w_i(t) = \kappa + \frac{e^{\xi t} \kappa (- \bar{a} r + \bar{a} (t-T_i)-(1+n)t + T_i) \bar{a})}{-r + \bar{a}}. \]

By maximizing the principal’s payoff over the threshold \( T^* \) we obtain

\[ T^* = -x_0 + \ln \left( \frac{\bar{a} - \kappa}{\kappa} \right) \frac{(1+n)\bar{a}}{\bar{a}}. \]

Now, we consider the case in which the IR constraint may bind. It follows from the proof of Proposition 4 that the bonus contract to agent \( i \) is given by \( w_i^*(T_i) \) for some experimentation time threshold \( T_i \).

The Lagrangian of the principal’s problem is

\[ \max_{(T_i, W_{i,0}) \alpha, \lambda, \mu} \left( \sum_{i=1}^{n} \left( -W_{i,0} + \int_{0}^{T_i} p_i (\pi - w_i^*(T_i)) a_{i,t} e^{-\int_{0}^{T_i} \bar{a}^a ds} dt \right) \right) \]

\[ + \lambda_i \left( \left. W_{i,0} + \int_{0}^{T_i} (p_i w_i^*(T_i) - \kappa) a_{i,t} e^{-\int_{0}^{T_i} \bar{a}^a ds} dt \right|_{T_i} - \bar{V} \right) + \mu_i W_{i,0} \right), \]

where \( \lambda_i \) is the multiplier associated to the constraint on each agent’s expected payoff and \( \mu_i \) is associated to the constraint \( W_{i,0} \geq 0 \).

The first order conditions with respect to \( W_{i,0} \) is

\[ -1 + \lambda_i + \mu_i = 0. \]

If \( W_{i,0} > 0 \) then \( \mu_i = 0 \) and \( \lambda_i = 1 \). The first order condition with respect to \( T_i \) is

\[ p_i \pi - \kappa = 0. \]

If \( \lambda_i \neq 0 \) and \( \mu_i \neq 0 \), we have \( T_i = T(\bar{V}) \).

If \( \lambda_i = 0 \) then \( T_i = T^* \).
A.4.1 Proof of Corollary 2

The derivative of $w_t$ with respect to $t$ is

$$e^{x_0}\kappa \left( -e^{nt\bar{a}}nr\bar{a} + e^{r(t-T)+(n-1)t+T}\bar{a}^2(r+(n-1)\bar{a}) \right) \frac{\bar{a} - r}{n\bar{a}}.$$ 

The numerator is negative iff $e^{(t-T)(r-\bar{a})} \leq 1 \iff r \geq \bar{a}$. Thus, $w_t$ increases in $t$.

The derivative of $w_t$ with respect to $r$ is given by

$$e^{-rT+x_0+(-1+n)t\bar{a}}\kappa \left( -e^{rT+\bar{a}} + e^{rT\bar{a}}(1-(t-T)(r-\bar{a})) \right) \frac{\bar{a}}{(r-\bar{a})^2},$$

which is negative iff $1 - e^{-(t-T)(r-\bar{a})} - (t-T)(r-\bar{a}) \leq 0$ which is always true as $1+x \leq e^x$ holds for every $x \in \mathbb{R}$. Thus, $w_t$ decreases in $r$.

To see that $w_t^*(T^*)$ increases in $\bar{a}$ note that the derivative of $w_t^*(T^*)$ with respect to $\bar{a}$, given that $T^*$ depends on $\bar{a}$, is given by

$$\left( \frac{\kappa r(-nt\bar{a}+nrT)+(n-1)r\bar{a}+(n-1)t+T^*)}{(r-\bar{a})^2} \right) e^{\frac{(n-1)t+T^*+(r-T^*)}{\bar{a}}}. $$

When $T^* = t$ the previous expression simplifies to $\frac{\kappa T^* (n\bar{a}-1)e^{t\bar{a}+x_0}}{\bar{a}}$. The term ** increases in $T^*$ since its derivative with respect to $T^*$ is given by

$$\kappa r \left( (-nt\bar{a}+nrt+1) e^{(T^*-t)(r-\bar{a})} - 1 \right) \frac{1}{\bar{a}(r-\bar{a})} > 0.$$ 

To see that each agent’s payoff increases in $\bar{p}$ note that $x_0$ decreases in $\bar{p}$ and that the derivative of an agent’s bonus at time $t$ with respect to $x_0$ is given by

$$\frac{\kappa e^{x_0} e^{nt\bar{a}} \left( (n+1)r-(n\bar{a}+r) e^{t-T)(r-\bar{a})} \right)}{(n+1)(r-\bar{a})} > 0.$$
A.4.2 Proof of Corollary 5

Let’s see that the expected bonus conditional on the event that there is a breakthrough is decreasing in $\bar{p}$. The expected bonus is given by

$$\int_0^{T^*} \bar{a}e^{-\bar{a}nt} \cdot dt \cdot (1 - \bar{p}) = (1 - \bar{p}) \cdot \frac{e^{-x_0} - e^{-nT^*\bar{a} - x_0}}{n}.$$ 

Thus, the expected bonus conditional on a success simplifies to

$$\frac{\kappa \bar{a}e^{-rT^*} \left( (n\bar{a} + r) (e^{rT^*} - e^{T^*\bar{a}}) e^{nT^*\bar{a} + x_0} + (r - \bar{a}) (e^{T^*\bar{a}} - 1) \right) - \left( r - \bar{a} \right) \left( e^{T^*\bar{a}} - 1 \right)^2}{(r - \bar{a}) (e^{T^*\bar{a}} - 1)}.$$

Note that $x_0 = \log \left( \frac{1 - \bar{p}}{\bar{p}} \right)$ is decreasing in $\bar{p}$ and $T^*$ is increasing in $\bar{p}$. Thus, to establish the claim we show that (29) decreases in $T^*$. Consider first the first term in the parentheses in the numerator of (29): $$\frac{\kappa \bar{a}e^{e^{rT^*} - e^{T^*\bar{a}}} \left( e^{nT^*\bar{a}} - e^{T^*\bar{a}} \right) - \left( e^{T^*\bar{a}} - 1 \right)^2}{(r - \bar{a}) (e^{T^*\bar{a}} - 1)}.$$

Taking the derivative of the term while ignoring factors that do not depend on $T^*$, we obtain

$$\frac{\kappa \bar{a}e^{-rT^*} \left( (n+1)T^*\bar{a} \right) \left( r - e^{-nT^*\bar{a}} \right) + \left( -ne^{T^*\bar{a}} \right) \left( e^{T^*\bar{a}} - e^{T^*\bar{a}} \right) + r}{(r - \bar{a}) (e^{T^*\bar{a}} - 1)^2},$$

which is zero at $T^* = 0$ and is negative for $T^* > 0$. To see this, note that the derivative of the term in parenthesis in the numerator is given by $n\bar{a} (\bar{a} - r) \left( -e^{-(n+1)T^*\bar{a}} \right) \left( e^{T^*\bar{a}} - e^{T^*\bar{a}} \right)$, which is strictly negative for $T^* > 0$ if and only if $r \geq \bar{a}$. This shows that the first term in the parenthesis in equation (29) is decreasing in $T^*$.

Consider the derivative of the second term in equation (29), which ignoring factors that do not depend on $T^*$, can be written as

$$\frac{e^{-rT^*} \left( e^{T^*\bar{a}} - 1 \right) - \left( e^{T^*\bar{a}} - 1 \right) e^{T^*\bar{a}}}{(e^{T^*\bar{a}} - 1)^2}.$$ 

This expression is zero at $T^* = 0$. It is negative for $T^* > 0$ since the derivative of the term in
parenthesis in the numerator is \( n\bar{a} (e^{T} - 1) (n\bar{a} + r) (-e^{nT\bar{a}}) < 0. \)

Thus, we have established that (29) decreases in \( T^* \) which completes the proof.

### A.5 Extensions and comparative statics in a one-task project

In this section we provide results for the case \( n \geq 2 \) that show how the optimal contract varies as the number of agents increases. We also provide some comparative statics on the optimal contract based on the

**Lemma 1** (Number of agents). *As the number of agents increases while keeping the total capacity \( n\bar{a} \) fixed, the agents wages converge uniformly to \( \kappa/p_t \) and the amount of experimentation converges to the first best. If \( \bar{U} = 0 \) the principal’s payoff increases in \( n \), whereas, if \( \bar{U} > 0 \) her payoff is single-peaked in \( n \).*

Figure 6 illustrates how optimal contract varies in \( n \geq 0 \) while keeping the total capacity, \( n\bar{a} \), remains constant.

If the outside option is greater than zero, hiring more agents is not always profitable, as for sufficiently many agents the sum of the outside options of all agents will surpass the value of the breakthrough \( \pi \). \( T(\bar{U}) \) defined in equation (5) does not depend on the number of agents. However, the efficient experimentation threshold \( \bar{T} \) does depend on \( n \). The agents’ payoff decreases in the number of agents. Thus, for every \( \bar{U} \) there is a sufficiently large number of agents such that the principal must provide a positive transfer \( W_0 \) at time zero. As \( \bar{T} \) increases the bonuses increase, while the amount of work that each agent performs decreases. The principal’s payoff is single-peaked in the number of agents, and there is an optimal number of agents to include in the project.

![Figure 6: Optimal bonus contracts for different numbers of agents keeping the total capacity fixed. Parameter values: \((\kappa, \bar{a}, \bar{p}, \pi, r, n\bar{a}, \bar{U}) = (1/4, 1, 9/10, 1, 0.5, 3, 0)\). As the number of agents increases agents receive less rents.](image-url)

52
From the expression for $w^*_1$ in Proposition 2, we can derive the following comparative statics.

The prior probability of success $\bar{p}$ is related to the riskiness of the project. For $\bar{p}$ close to one, for example, the variance of the revenues from the project increases as $\bar{p}$ decreases. Corollary 2 shows that projects with a lower prior probability $\bar{p}$ give higher bonuses to the agents. Thus, given two projects that differ in $\bar{p}$ and $\pi$ such that they have the same experimentation threshold $T^*$, the expected bonus conditional success is higher in the riskier project. Furthermore, in Corollary 5 below we show that, fixing all other variables, conditional on a breakthrough, the expected discounted bonuses are higher when $\bar{p}$ is smaller. This result is in contrast with the risk-incentives trade-off found in Holmstrom (1979). Empirical tests of this trade-off have yielded mixed results (Prendergast (2000) and Prendergast (2002)).

**Corollary 5** (Risk and incentives). *If the IR constraint does not bind the expected discounted bonus conditional on there being a success is decreasing in $\bar{p}$.*

Consider two projects with different $\bar{p}$ and $\pi$ such that they give the same expected payoff to the planner.\(^{50}\) Corollary 5 implies that the project with lower $\bar{p}$ will give higher expected bonuses, conditional on a bonus being paid.

### A.6 Optimal disclosure

**Proof of Proposition 3:**

We assume that breakthroughs are verifiable by all the agents and that disclosures fully reveal that a breakthrough occurred. The principal observes all breakthroughs but players observe only their own breakthroughs. The space of histories $\mathcal{H}^t$ contains all breakthroughs by all players that have been obtained and all previous disclosures. A disclosure $d$ is a function from histories $\mathcal{H}^t$ to $\{0, 1\}^I$ with $d(h'_i) = 1$ if the principal reveals a breakthrough to agent $i$ after history $h'_i$ and $d(h'_i) = 0$ otherwise. Because breakthroughs are verifiable the principal can only disclose breakthroughs that have indeed taken place. A disclosure policy must be adapted to the $\sigma$-algebra of histories.

The space of possible disclosure policies is very large. Some examples of disclosure policies are: disclose a discovery as soon as it occurs with probability one, disclose a discovery two seconds after it occurs with probability $q$ and then disclose at Poisson rate $\lambda$ after that, not disclose a breakthrough if it arrives before some time $\tilde{t}$ and disclose it right away thereafter.

The problem of choosing the optimal disclosure policy can be simplified by making the following two observations. First, since experimentation on task 1 after the first breakthrough has no value to the principal, she will stop an agent’s experimentation as soon as she reveals to him

\(^{50}\)The same argument applies for two projects with the same expected payoff under the optimal contract.
that a breakthrough arrived. In addition, the principal has to at least compensate the agent for the cost of effort. Therefore, allowing an agent to experiment after he knows a breakthrough has been achieved increases the cost of providing incentives before and after a breakthrough.

The second observation is that from the viewpoint of each agent $i$ the disclosure policy translates to a measurable process $\int_0^t \tilde{d}_{i,s} ds \leq \int_0^t a_{i,s} ds$, as a function of $t$, where $\tilde{d}_{i,s} e^{\int_0^s \tilde{d}_{i,t} ds}$ is the probability density that there is a disclosure at time $t$ given that the task is good and, hence, agent $i$ is not allowed to experiment any further. This density combines the probability that a breakthrough is realized and that it is revealed. If the principal discloses breakthroughs at the first opportunity $\tilde{d}_{i,s}$ coincides with $a_{i,s}$. If principal does not disclose other players’ breakthroughs then $\tilde{d}_{i,s} = 0$. It may also be that $\tilde{d}_{i,s} > a_{i,s}$ at times in which the principal concentrates disclosures with higher probability. Finally $\int_0^t \tilde{d}_{i,s} ds$ is allowed to jump as a function of the history. Jumps occur when there is a positive probability of disclosure at a given time.

Proposition 1 characterizes the wage $i$ must receive given the process $\int_0^t \tilde{d}_{i,s} ds$. Let $T_i$ denote the supremum of the times at which agent $i$ exerts positive effort. Solving the differential equation given by equation (2) in Proposition 1 yields

$$w_{i,t} = \kappa \left( e^{-\int_t^{T_i} (r - a_{i,t}) dt + \int_0^t (a_{i,s} + \tilde{d}_{i,s}) ds + x_0 } + 1 \right) + e^{rt + \int_0^t \tilde{d}_{i,s} ds} \int_t^{T_i} \kappa e^{-\tau + \int_0^\tau \tilde{d}_{i,s} ds + x_0 } d\tau.$$

Replacing the expression for $w_{i,t}$, $i$’s expected payoff becomes

$$\tilde{p} e^{x_0} \int_0^{T_i} a_{i,t} \left( \int_t^{T_i} \kappa e^{\int_t^\tau a_{i,s} ds - \tau} d\tau + \kappa e^{rt} \left( e^{-\int_t^{T_i} (r - a_{i,t}) dt } - 1 \right) \right) dt = \int_0^{T_i} a_{i,t} \left( \int_t^{T_i} \kappa a_{i,s} e^{\int_t^\tau a_{i,s} ds - r \tau} d\tau \right) \tilde{p} e^{x_0}.$$

Thus, $i$’s instantaneous and expected payoff does not depend on the process $\int_0^t \tilde{d}_{i,s} ds$. Changing the disclosure policy affects the agent’s optimal wage but also his belief update about receiving a reward. These effects precisely cancel so that the agent’s expected payoff does not depend on

---

51 If the principal reveals a breakthrough that occurred at time $t$, at time $s$ at rate $\lambda_s^i$ (where $\lambda_s^i$ may be infinite, representing a deterministic disclosure), then the probability that $i$ obtains a breakthrough at time $\tau$ is

$$a_{i,\tau} \tilde{p} e^{-\int_0^\tau a_{i,s} ds - \int_0^\tau a_{-i,s} ds } + \tilde{p} e^{-\int_0^\tau a_{i,s} ds } a_{i,\tau} \int_0^\tau a_{i,s} e^{-\int_0^\tau \lambda_s^i ds - \int_0^\tau a_{-i,s} ds } d\tilde{s} = a_{i,\tau} \tilde{p} e^{-\int_0^\tau a_{i,s} ds - \int_0^\tau \tilde{d}_{i,s} ds },$$

with $\tilde{d}_{i,s} = \left( \frac{d}{d\tau} \log \left( e^{-\int_0^\tau a_{i,s} ds - \int_0^\tau a_{-i,s} ds } \right) \right)^{-1}$.

52 In the proof of Proposition 1 we do not impose restrictions on $\int_0^0 a_{-i,s} ds$, therefore, it can be replaced by any other process.
the disclosure, conditional on a level of experimentation. This observation implies that the agent’s payoff does not depend on the choice of disclosure policy under the optimal contract.

We are now ready to prove that the principal can never gain from not disclosing right away in the one-task project. Suppose the principal chooses disclosure policy $\int_0^t \tilde{d}_{i,s} ds$ and that each agent $i$’s effort function is given by $a_{i,t}$. The principal’s payoff from agent $i$’s work can be written as

$$
\int_0^T \left( p_t \pi e^{-\int_0^t p_s a_s ds} - \kappa \left( \bar{p} e^{-\int_0^t (a_{i,s} + \tilde{d}_{i,s}) ds} + (1 - \bar{p}) \right) \right) a_{i,t} e^{-rt} dt
$$

The last integral in the previous expression corresponds to the agent’s payoff and does not depend on the disclosure policy. However, the first integral does and it is increasing in $\int_0^t \tilde{d}_{i,s} ds$. When an agent works after another agent has found a discovery, the principal has to compensate the agent for the cost of effort but does not gain anything, in reduced costs, from the duplicated effort.\textsuperscript{53,54}

AGENT’S DISCLOSURE DECISION:

We now consider the agent’s disclosure problem. We assume that the agents are able to conceal the achievement of a breakthroughs, but that a breakthrough is verifiable once announced. In the optimal contract with observable breakthroughs, the bonus increases in the time at which a breakthrough is reached. Therefore, an agent may find it profitable to delay the disclosure of a privately observed discovery in order to receive a higher bonus. Delaying disclosure is costly because agents discount future bonuses and because another agent may succeed while disclosure is delayed. We show that at the optimal contract these costs overcome the benefits from an increased bonus and, thus, delaying disclosure is not optimal. In fact, the expected payoff of delaying disclosure until time $t$ from time $\tilde{t}$ is given by $w_{i,t} e^{-(n-1)\bar{a}r(t-\tilde{t})}$, as the bonus at time $t$ is $w_{i,t}$ and with probability $e^{-(n-1)\tilde{a}(t-\tilde{t})}$ other agents do not obtain a success in the interval $[\tilde{t},t]$. This expected payoff decreases over time since

$$
\frac{\partial w_{i,t} e^{-(n-1)\bar{a}t - rt}}{\partial t} = \kappa \left( -e^{-t((n-1)\bar{a}+r)} \right) (r e^{nt\tilde{a} + x_0} + 1 + (n-1)\tilde{a}) < 0.
$$

The result is stated in the following Proposition.

\textsuperscript{53}If breakthroughs are not verifiable the principal may gain from sometimes claiming a breakthrough that did not occur so as to make the agents more optimistic about the project which lowers the cost of providing incentives. The payoff of the agent does depend on his belief about the feasibility of the project, therefore, the present argument does not apply if breakthroughs are unverifiable.

\textsuperscript{54}The result is true for symmetric and asymmetric agents.
Proposition 12 (Unobservable discoveries). Under the optimal contract of the one task project, agents do not delay the disclosure of privately observed discoveries.

This result extends immediately to the two-task costly-incentives case if agents are not able to begin the second task without the principal’s permission.

B Appendix: Two-task project

B.1 Second task: Proof of Proposition 4

Suppose that under the optimal contract each agent $i$ gets expected utility $V_i(h^1)$ after history $h^1$ in the first task. We will see that it is optimal for the principal to offer a contract of the form given by equation (7) and specifying that agents work at maximum speed until a time threshold. Suppose that the principal offers a contract that does not satisfy equation (7) and implements efforts $\{a_{i,t}\}_{i,t}$ for each agent $i$. From Proposition 1 the contract that satisfies (7) is the least costly contract that implements these efforts. Thus, the principal is weakly better off by offering such a contract.

It follows from the proof of Proposition 2 that the principal implements the maximum effort before the deadline. Shifting effort from interval $[t + dt, t + 2dt]$ to interval $[t, t + dt]$ has a second order effect on the principal’s payoff and a third order effect on the agent’s payoff. Thus, the principal can be made weakly better off by implementing effort at the maximum while maintaining the agent’s utility at the same level.

B.2 First task. The agents’ problem: proof of Proposition 5

As before we solve the agent’s problem using optimal control. As in the proof of Proposition 1 we can write the Hamiltonian as

$$H_t = (w_{i,t}^1 + v_{i,t}^1 - \kappa^1) e^{-\gamma^1 t} a_{i,t}^1 \bar{p}^1 - \kappa^1 a_{i,t}^1 (1 - \bar{p}^1) + \sum_{j \neq i} v_{i,t}^j a_{j,t}^1 e^{-\gamma^1 t} \bar{p}^1 + \eta_i \sum_i a_{i,1}^1,$$

where $y_t^1 = \int_0^t a_{i,t}^1$, $v_{i,t}^j$ denotes the expected payoff of agent $i$ in the second task when agent $j$ achieves a success at time $t$ and $\eta_{i,t}$ is the costate variable associated to $y_t^1$. By Pontryagin’s principle $\eta_i$ is absolutely continuous and evolves according to the following differential equation:

$$\dot{\eta}_i = r \eta_i + (w_{i,t}^1 + v_{i,t}^1 - \kappa^1) e^{-\gamma^1 t} a_{i,t}^1 \bar{p}^1 + \sum_{j \neq i} v_{i,t}^j a_{j,t}^1 e^{-\gamma^1 t} \bar{p}^1. \quad (32)$$
Define \( \tilde{\gamma}_{i,t} \) so that

\[
\eta_{i,t} = -\left( w_{i,t}^1 + v_{i,t}^l - \kappa^1 \right) e^{-\gamma_i^1 \tilde{p}^1} + \kappa^1 \left( 1 - \tilde{p}^1 \right) + \tilde{\gamma}_{i,t} \tilde{p}^1.
\]

Thus, \( \tilde{\gamma}_{i,t} > 0 \) implies \( a_{i,t} > 0 \) and \( \tilde{\gamma}_{i,t} < 0 \) implies \( a_{i,t} < 0 \). Replacing \( \tilde{\gamma}_{i,t} \) in equation (32) we obtain

\[
\dot{\tilde{\gamma}}_{i,t} = r \tilde{\gamma}_{i,t} - e^{-\gamma_i^1 \tilde{p}^1} \left( w_{i,t}^1 + v_{i,t}^l - \kappa^1 \right) \left( a_{i,t}^1 + \sum_{j \neq i} v_{i,j,t}^l a_{j,t}^1 e^{-\gamma_j^1 \tilde{p}^1} + \left( \dot{w}_{i,t}^1 + \dot{v}_{i,t}^l \right) e^{-\gamma_j^1 \tilde{p}^1} \right) + \kappa^1 \kappa t \tilde{p}^1.
\]  

(33)

Let \( T^1_i = \sup \{ t \in \mathbb{R}_+ | a_{i,t} > 0 \} \). We will see that the principal stops agents’ experimentation at a finite time therefore it is without loss to assume \( T^1_i < \infty \). If agents other than agent \( i \) work past time \( T^1_i \) and agent \( i \) participates in the second task then agent \( i \) has a positive expected payoff at time \( T^1_i \) associated to the rents obtained working in the second task, if another players succeeds. This payoff is the salvage value at time \( T^1_i \) and is given by \( G(y_{T_i}, T^1_i) \tilde{p}^1 \) where

\[
G(y_{T_i}, T^1_i) \equiv \int_{T_i}^{\infty} \int_{T_i}^{\infty} e^{-\gamma_i - f_{T_i}^1 a_{i,t} ds - rt} dt.
\]

The transversality condition is

\[
\eta_{i,T^1_i} e^{-r T^1_i} = \frac{\partial G(y_{T_i}, T^1_i)}{\partial y_{T^1_i}} \tilde{p}^1 = -\tilde{p}^1 \int_{T_i}^{\infty} \sum_{j \neq i} v_{i,j,t}^1 a_{j,t}^1 e^{-\gamma_j^1 t - f_{T_i}^1 a_{j,t} ds - rt} dt
\]

which, rewriting in terms of \( \tilde{\gamma}_{i,t} \) yields

\[
\tilde{\gamma}_{i,T^1_i} = \left( w_{i,T^1_i}^1 + v_{i,T^1_i}^l - \kappa^1 \right) e^{-\gamma_i^1 \tilde{p}^1} - \kappa^1 \kappa t \tilde{p}^1 - G(y_{T_i}, T^1_i) e^{\gamma_i^1 T^1_i}.
\]  

(34)

Let \( u_{i,t} = w_{i,t}^1 + v_{i,t}^l \) be the expected payoff of agent \( i \) when he achieves a breakthrough at time \( t \). Solving the differential equation for \( u_{i,t} \) and replacing condition (34) we obtain

\[
u_{i,t} = \int_{T_i}^{T^1_i} e^{T^1_i - f_{T_i}^1 (r + a_{i,t}^1) ds} \left( (\tilde{\gamma}_{i,T^1_i} + G(y_{T_i}, T^1_i) e^{T^1_i - f_{T_i}^1} \kappa^1 \kappa t \tilde{p}^1) + \kappa^1 \kappa t \tilde{p}^1 - e^{T^1_i - f_{T_i}^1 (r + a_{i,t}^1) ds} \sum_{j \neq i} v_{i,j,t}^1 a_{j,t}^1 ds + \int_{T_i}^{T^1_i} e^{T^1_i - f_{T_i}^1 (r + a_{i,t}^1) ds} \sum_{j \neq i} v_{i,j,t}^1 a_{j,t}^1 ds + \right)
\]

\[
\int_{T_i}^{T^1_i} e^{T^1_i - f_{T_i}^1 (r + a_{i,t}^1) ds} \sum_{j \neq i} v_{i,j,t}^1 a_{j,t}^1 ds + \int_{T_i}^{T^1_i} e^{T^1_i - f_{T_i}^1 (r + a_{i,t}^1) ds} \sum_{j \neq i} v_{i,j,t}^1 a_{j,t}^1 ds + \right) \left( \tilde{\gamma}_{i,s} - r \tilde{\gamma}_{i,s} - \kappa^1 \kappa t \tilde{p}^1 \right) ds.
\]

57
Integrating by parts and simplifying yields

\[ u_{i,t} = e^{-\int_t^{T_i} (r + a_{i,t}^1) ds} y_i^1 + \kappa^1 + \sum_{j \neq i} \int_t^{T_j} e^{-\int_j^{T_i} (r + a_{j,s}^1) ds} v_{i,s} a_{j,s}^1 + \sum_{j \neq i} \int_t^{T_i} e^{-\int_j^{T_i} (r + a_{i,s}^1) ds} v_{j,s} a_{i,s}^1 + \sum_{j \neq i} \int_t^{T_i} e^{-\int_j^{T_i} (r + a_{i,s}^1) ds} (\gamma_i, T_j - T_i) e^{-\int_j^{T_i} (r + a_{i,s}^1) ds} + \gamma_i \]  

Equation (35) represents a first order condition for the agent’s choice of effort. As in the one-task case, \( u_{i,t} \) is minimized by setting \( \gamma_i = 0 \). Replacing \( \gamma_i = 0 \) into equation (33) yields the expression for \( u_{i,t}^{\min} \) in equation (9). An argument exactly analogous to the one that establishes that the necessary conditions are sufficient in the one-task case (page 46) also applies to this setting. Thus, each agent \( i \)’s choice when facing contract \( u_{i,t}^{\min} \) in equation (9) is effort function \( (a_{i,t})_{t \geq 0} \).

**B.3 Derivations for section 4.2**

Consider the principal’s decision whether to shift effort from one instant to the next. Using a second degree Taylor expansion, agent \( i \)’s expected payoff at time \( t \) can be written as

\[ \tilde{V}_{i,t} = V_{i,t} + \frac{1}{2} dt^2 v_{i,t}^2 (pa_{-i,t})^2 + dt(pa_{-i,t}v_{i,t}) + dt(\frac{1}{2} dt^2 (pa_{-i,t}^2 + a_{-i,t}) + dt(\gamma_i, T_i - T_i) e^{-\int_t^{T_i} (r + a_{i,s}^1) ds} + \gamma_i \]  

where \( V_{i,t} \) is given by equation (16), replacing \( w_{i,t} \) by \( u_{i,t} \). Thus, we obtain

\[ -\frac{\partial \tilde{V}_{i,t}}{\partial a_{i,t}} + \frac{\partial \tilde{V}_{i,t}}{\partial a_{i,t + dt}} = -\frac{\partial V_{i,t}}{\partial a_{i,t}} + \frac{\partial V_{i,t}}{\partial a_{i,t + dt}} + \left( p_i a_{-i,t} v_{i,t} \right) dt^2 + o(dt^2) \]  

**B.4 First task: the principal’s problem**

We now state the principal’s problem as an optimal control problem given the continuation contract derived in Proposition 4 and prove the Propositions in section 4.

**The principal’s problem** We start by describing the constraints that the principal faces when designing the first-task contract. In section 5 we saw that an agent’s choice of effort satisfies
equation (33). Additionally the limited liability constraint implies that

\[ u_{i,t} - v^i_{i,t}(T^2(i,t)) \geq 0. \]  

That is, the payoff the agent gets after succeeding at time \( t \) is at least the continuation payoff from experimenting in task 2.

When stating the principal’s problem we let the multiplier \( \tilde{y}_{i,t} \) in the agent’s necessary condition be controlled by the principal so as to maximize her payoff. It is without loss to assume \( \tilde{y}_{i,t} \geq 0 \) since for any contract with \( \tilde{y}_{i,t} < 0 \) for \( t \) in some set \( \Theta \) there is a payoff equivalent contract that gives the same incentives to the agents with \( \tilde{y}_{i,t} = 0 \) for \( t \in \Theta \). Also, as long as \( u_{i,t} > v^i_{i,t}(T^2(i,t)) \) the principal sets \( \tilde{y}_{i,t} = 0 \). If not, by decreasing \( \tilde{y}_{i,t} \) slightly the principal can give incentives for the same effort choice at a lower cost.

From the agents’ problem we have the following additional constraints:

\[ \tilde{y}_{i,t}(a^1_{i,t} - \bar{a}) \geq 0 \]  
\[ \tilde{y}_{i,t}a^1_{i,t} \geq 0, \]  

which imply that when \( \tilde{y}_{i,t} > 0 \), \( a_{i,t} = \bar{a} \) and when \( \tilde{y}_{i,t} < 0 \), \( a_{i,t} = 0 \).

The state variables are \( y^1_t = \int_0^t a^1_s ds, u_{i,t}, \) and \( \tilde{y}_{i,t} \). The control variables at time \( \tau \) are the second task time thresholds \( T^2(k, \tau) = (T^2_1(k, \tau), T^2_2(k, \tau), \ldots, T^2_n(k, \tau)) \), each agent’s effort \( a^1_{i,\tau} \) and \( \tilde{y}_{i,\tau} \) which we denote \( d_{i,\tau} \). Let \( T^1_t = \sup \{ t \in \mathbb{R}_+ | a_{i,t} > 0 \} \) and \( T^1 = \max_i T^1_t \). The differential equations for the state variables are

\[ \dot{y}^1_t = \sum_i a^1_{i,t}, \]  
\[ \dot{u}_{i,t} = \left( (u_{i,t} - \kappa^1)(a_{-i,t} + r) - \sum_{j \neq i} v^1_{i,j}a^1_{j,t} - \kappa^1 r e^{\gamma^1} + d_{i,\tau} e^{\gamma^1} \right), \]  
\[ \dot{\tilde{y}}_{i,t} = d_{i,t}. \]

We introduce also a variable related to the lump-sum transfer at time zero. Define the state variable \( \tilde{W}_{i,t} \) for each \( t \) and \( i \) such that \( \tilde{w}_{i,t} = \omega_{i,t} \) with \( \omega_{i,t} \) a control variable. The constraints are \( \tilde{W}_{i,0} = 0 \) and \( \tilde{W}_{i,T^1} \geq 0 \).

In order to include the IR constraint in the optimal control problem we introduce the new variable:

\[ \bar{U}_{i,\tau} = \int_0^\tau \left[ \left((u_{i,t} - \kappa^1) e^{-\gamma^1} \bar{p}^1 - (1 - \bar{p}^1) \kappa^1 \right) a^1_{i,t} e^{-\gamma t} + \sum_{j \neq i} v^1_{i,j}(T^2(j,t)) a^1_{j,t} e^{-\gamma^1 - \gamma t} \bar{p}^1 + \omega_{i,t} \right] dt. \]
Its law of motion is \( \dot{U}_{i,t} = \left[ (u_{i,t} - \kappa^1) a^1_{i,t} e^{-\gamma_1^1 - rt} \tilde{p}^1 - (1 - \tilde{p}^1) \kappa^1 + \sum_{j \neq i} v^j_{i,t} (T^2(j,t)) a^1_{j,t} e^{-\gamma_1^1 - rt} \tilde{p}^1 \right] \) with \( \tilde{U}_{i,0} = 0 \). For the IR constraint to hold we must \( \tilde{U}_{i,T^1} \geq \tilde{U}_i \).

The expected transfer from breakthroughs, as agents exert maximum effort task from Proposition 4, is given by

\[
\pi(T^2(k, \tau)) = \sum_i \int_0^{T^2_i(k, \tau)} \rho_i^2 \pi^2 e^{-\gamma^2_i} \rho^2_0 \sum ds \ dt + \pi^1.
\]

The cost incurred by the agents in the second stage if the vector of stopping times is \( T^2(k, \tau) \) is given by

\[
c(T^2(k, \tau)) = \sum_i \int_0^{T^2_i(k, \tau)} \kappa^2 \rho^2 e^{-\gamma^2_i} \rho^2_0 ds \ dt.
\]

The Hamiltonian is given by

\[
H^R = \sum_i \left( \pi(T^2(i,t)) - c(T^2(i,t)) - u_{i,t} - \sum_{j \neq i} v^j_{i,t} (T^2(i,t)) a^1_{i,t} e^{-\gamma^1_i - rt} \tilde{p}^1 - \omega_{i,t} + \alpha_i \sum_i a^1_{i,t} \right.
\]
\[
+ \sum_i \eta_{i,t} (u_{i,t} - \kappa^1) (a^1_{i,t} + \rho^1) \eta_{i,t} (a^1_{i,t} - \tilde{a}) + \beta^2_{i,t} \tilde{a} a^1_{i,t} + \tilde{\xi}_{i,t} (u_{i,t} - v^1_{i,t})
\]
\[
+ \sum_i \xi_{i,t} \left[ (u_{i,t} - \kappa^1) a^1_{i,t} e^{-\gamma^1_i} \rho^1 - (1 - \tilde{p}^1) \kappa^1 \right] a^1_{i,t} e^{-\gamma^1_i - rt} \tilde{p}^1 + \omega_{i,t} 
\]
\[
+ \sum_i v^j_{i,t} (T^2(j,t)) a^1_{j,t} e^{-\gamma^1_j - rt} \tilde{p}^1 + \omega_{i,t}
\]

where \( \omega_i \) is the costate variable associated to \( y_i \), \( \lambda_{i,t} \) is the costate associated to \( u_{i,t} \), \( \eta_{i,t} \) is the costate associated to \( \tilde{y}_{i,t} \), \( \xi_{i,t} \) is the costate associated to the IR constraint, \( \eta_{i,t} \) is the multiplier associated to \( \tilde{W}_{i,t} \) and \( \beta_{i,1} \) for \( m \in \{1, 2\} \) and \( \tilde{\xi}_{i,t} \) are multipliers associated to the constraints.

**Evolution of co-state variables** By Pontryagin’s principle

\[
\dot{\pi} = -\frac{\partial H^R}{\partial y^1_i} = \sum_i \left( \pi(T^2(i,t)) - c(T^2(i,t)) - u_{i,t} - \sum_{j \neq i} v^j_{i,t} (T^2(i,t)) a^1_{i,t} e^{-\gamma^1_i - rt} \tilde{p}^1 + \lambda_{i,t} \kappa^1 e^{\gamma^1_i} + \pi^1 \right)
\]
\[
+ \rho_{i,t} e^{\gamma^1_i} + \delta_{i,t} \left[ (u_{i,t} - \kappa^1) a^1_{i,t} e^{-\gamma^1_i - rt} \tilde{p}^1 + \sum_{j \neq i} v^j_{i,t} (T^2(j,t)) a^1_{j,t} e^{-\gamma^1_j - rt} \tilde{p}^1 \right] \right).
\]
\[ \lambda_{i,t} = -\frac{\partial H_{RR}}{\partial u_{i,t}} = (1 - \xi_{i,t})a_{i,t}e^{-\gamma_{i,t} - \beta_{1,i} - \lambda_{i,t} (a_{i,t} + r)} - \xi_{i,t}. \] (43)

\[ \hat{\eta}_{i,t} = -\frac{\partial H_{RR}}{\partial \hat{y}_{i,t}} = r\lambda_{i,t}e^{\gamma_{i,t}} - \beta_{1,i}^{1}(a_{i,t} - \bar{a}) - \beta_{1,i}^{2}a_{i,t}. \] (44)

\[ \dot{\xi}_{i,t} = 0. \] (45)

\[ \dot{\nu}_{i,t} = 0 \] (46)

From the agent’s problem we have the following constraint at \( T_{i}^{1} \):

\[ h_{i}(y_{T_{i}}, T_{i}) = -u_{i,T_{i}^{1}} + \kappa_{i}^{1}(1 + e^{\gamma_{T_{i}^{1}} + x_{0}}) + G(y_{T_{i}^{1}}, T_{i})e^{\gamma_{T_{i}^{1}} + rT_{i}^{1}} + \bar{y}_{i,T_{i}^{1}}e^{\gamma_{1}}. \] (47)

Let \( \hat{\mu} \) be the multiplier associated with this constraint. The transversality conditions of the principal’s problem are then

\[ \lambda_{i,0} = 0, \quad \lambda_{i,T_{i}^{1}} = \frac{\partial h_{i}}{u_{i,T_{i}^{1}}} = -\hat{\mu}_{i}, \quad \eta_{i,0} = 0, \quad \eta_{i,T_{i}^{1}} = \frac{\partial h_{i}}{\hat{y}_{i,T_{i}^{1}}} = \hat{\mu}_{i}e^{\gamma_{T_{i}^{1}}}, \] (48)

\[ \alpha_{T_{i}^{1}} = \frac{\partial h_{i}}{y_{T_{i}^{1}}} = \hat{\mu}_{i} \left( \bar{y}_{i,T_{i}^{1}}e^{\gamma_{1}^{1}} + e^{\gamma_{1}^{1} + x_{0}} \kappa_{1} \right) + \sum_{j \neq i} \hat{\mu}_{j} \left( \bar{y}_{j,T_{i}^{1}}e^{\gamma_{1}^{j}} + e^{\gamma_{1}^{j} + x_{0}} \kappa_{1} \right) \cdot 1\{T_{i}^{1} = T_{j}^{1}\}, \text{ and } \nu_{i,T_{i}^{1}}, \xi_{i,T_{i}^{1}} \geq 0. \]

We must also have \( \nu_{i,T_{i}^{1}}(U_{i} - \tilde{U}) = 0 \) and \( \nu_{i,T_{i}^{1}} \tilde{W}_{i,T_{i}^{1}} = 0 \)

From conditions (45) and (46) we can write \( \xi_{i,t} = \xi_{i} \) and \( \nu_{i,t} = \nu_{t} \) for each player \( i \).

**Maximization with respect to \( \omega_{i,t} \)** If \( \omega_{i,t} > 0 \) for some time \( t \) we must have

\[ -1 + \xi_{i} + \nu_{i} = 0. \]

Thus, if the lump-sum at time zero \( \tilde{W}_{i,T_{i}^{1}} > 0 \) then \( \nu_{i} = 0 \) and \( \xi_{i} = 1 \).

**Maximization with respect to \( d_{i,t} \)** In order to solve the principal’s problem we will assume that the derivative of the agent’s multiplier \( \hat{y}_{i,t} \) is bounded by a large constant \( M \). It is clear that there is \( M > 0 \) such that it is not profitable for the principal to set the slope of \( \hat{y}_{i,t} \) above \( M \). From the agent’s necessary condition, a large slope implies a large bonus for success. Whenever \( |d_{i,t}| \neq M \), since \( H_{RR}^{i} \) maximizes \( d_{i,t} \), the terms that multiply \( d_{i,t} \) must be zero. Thus,

\[ \lambda_{i,t}e^{\gamma_{i,t}} + \eta_{i,t} = 0. \] (49)
Suppose \( \tilde{\gamma}_{i,t} > 0 \) and \( a_{i,t} = \bar{a} \) for \( t \) in a time interval \([t_0, t_1]\) then \( \beta_{i,t}^1(a_{i,t} - \bar{a}) - \beta_{i,t}^2 a_{i,t}^1 = 0 \) for \( t \in [t_0, t_1] \). Differentiating equation (49) with respect to \( t \) and combining it with (44) we obtain

\[
- (a_{i,t}^1 + \bar{a}_{-i,t}^1) \lambda_{i,t} - \dot{\lambda}_{i,t} = r \lambda_{i,t},
\]

and thus, \( \lambda_{i,t} = \lambda_{i,0} e^{-\int_{t_0}^t (r + a_t) ds} \) for \( t \in [t_0, t_1] \). From the transversality condition, \( \lambda_{i,0} = 0 \), if \( \tilde{\gamma}_{i,t} > 0 \) for every \( t \leq T_i^1 \) we must have \( \lambda_{i,t} = 0 \) for all \( t \leq T_i^1 \).

Replacing into equation (43) we obtain

\[
\tilde{\xi}_{i,t} = a_{i,t}^1 e^{-y_{i,t}^1 + \bar{p}^1} + a_{i,t}^1 \lambda_{i,0} e^{-\int_{t_0}^t (r + a_t) ds} - \xi_{i,t} \lambda_{i,t}^1 e^{-y_{i,t}^1 + \bar{p}^1},
\]

if \( \tilde{\gamma}_{i,t} > 0 \) for all \( t \) in \([t_0, t_1]\). These derivations are useful for the characterization of the cheap and intermediate incentive costs cases.

**Maximization with respect to \( T_i^2(k, \tau) \)** For the maximization with respect to \( T_i^2(i, \tau) \) note that if \( \tilde{\xi}_{i, \tau} = 0 \), that is when \( u_{i,t} > v_{i,t}^i \) then the \( T_i^2(i, \tau) \) solves

\[
\max \int_0^{T_i^2(i, \tau)} \bar{a} (p_i^2 \pi^2 \kappa^2) e^{-\int_{t_0}^t (r + a_t^2) ds} dt,
\]

which corresponds to the planner’s problem and therefore corresponds to the efficient belief threshold for \( i \). As it is unprofitable to let agents continue experimentation once the efficient stopping belief is reached, it follows that \( T_i^2(i, \tau) \geq T_k^2(i, \tau) \) for each player \( k \) and timing of success \( \tau \).

If \( \tilde{\xi}_{i, \tau} > 0 \) then \( T_i^2(i, \tau) \) solves

\[
\max \left( \int_0^{T_i^2(i, \tau)} \bar{a} (p_i^2 \pi^2 \kappa^2) e^{-\int_{t_0}^t (r + a_t^2) ds} dt \right) \cdot a_{i,t}^1 e^{-y_{i,t}^1 + \bar{p}^1} - \tilde{\xi}_{i, \tau} v_{i,t}^i.
\]

If \( \tilde{\gamma}_{i, \tau} > 0 \) for \( \tau \) in time interval \([t_0, t_1]\) replacing \( \tilde{\xi}_{i, \tau} \) from equation (50) and \( \lambda_0^2 \) the first order condition is given by

\[
\left( (\pi^2 - \kappa^2) e^{-\sum T_k^2(i, \tau) a - x_0^2 + \bar{p}^1} \right) \bar{p}^1 - \left( \bar{p}^1 (1 - \zeta_i) + \lambda_{i,0} e^{\int_{t_0}^t (r + a_t) ds} \right) \left( e^{\delta T_i^2(i, \tau)} - 1 \right) \kappa^2 (1 - \bar{p}^2) = 0.
\]

If \( T_k^2(i, \tau) \) does not depend on \( \tau \) then \( T_i^2(i, \tau) \) does not depend on \( \tau \). If \( \lambda_{i,0} = 0 \) and the IR constraint does not bind (\( \zeta_i = 0 \)), the threshold satisfies \( 2T_i^2(i, \tau) + \sum_k T_k^2(i, \tau) = -\frac{\bar{p}^1 + \log(\pi^2 - \kappa^2)}{\delta} \). That is, \( T_i^2(i, \tau) \) is the principal’s optimal one-task project threshold given that agents other than \( i \) have thresholds \( T_k^2(i, \tau) \). Thus, if \( \tilde{\gamma}_{i,t} > 0 \) for every \( t \leq T_i^1 \) and every agent \( i \) then each agent \( i \)’s threshold is the one-task optimal threshold for every \( t \).
where \( T \) is the state variables and \( f \) variables). Denote \( f \) through (41) and denote the integrand in the previous expression , where \( x \), \( a \), and suppose \( a_{i,\tau}^1 > 0 \). The derivative of the agent \( i \)'s payoff with respect to \( T_i^2(k, \tau) \) can be computed from the wage equation (7) and is given by

\[
e^{-T_i^2(k, \tau)r} \left( e^{T_i^2(k, \tau)a} - 1 \right) \kappa^2 a(1 - \bar{p}^2).
\]

\( T_i^2(k, \tau) \) maximizes \( H_i^{RR} \). The first order condition with respect to \( T_i^2(k, \tau) \) for \( k \neq i \), \( T_k^2(k, \tau) \geq T_i^2(k, \tau) \), is given by

\[
(\pi^2 - \kappa^2) \bar{p}^2 e^{-T_i^2(k, \tau)(r + na)} - \bar{p}^2 \int_{T_i^2(k, \tau)}^{T_k^2(k, \tau)} (\pi^2 - \kappa^2) e^{-y_i^2 - rs} ds - (1 - \bar{p}^2) e^{-T_i^2(k, \tau)r} \kappa^2 \quad (53)
\]

\[-(1 - \varphi) e^{-T_i^2(k, \tau)r} \left( e^{T_i^2(k, \tau)a} - 1 \right) (1 - \bar{p}^2) \kappa^2 - \lambda_i \kappa e^{y_i^2 + rt} e^{-T_i^2(k, \tau)r} \left( e^{T_i^2(k, \tau)a} - 1 \right) (1 - \bar{p}^2) \kappa^2 \frac{1}{\bar{p}^1} + = 0.
\]

**Existence of a solution to the principal’s problem**  In order to prove existence of a solution we will re-write the problem so that the principal’s payoff is in terms of the expected payoff of each agent in the optimal contract. Denote

\[
W_i^2(v_i) = \int_0^{T_i(v_i)} \left( p_i^2 \pi^2 - \kappa^2 \right) a_{i,t}^2 e^{-\int_0^t p_i^2 a_t^2 - rt} dt,
\]

where \( T_i(v_i) \) is the threshold that gives the agent expected utility \( v_i \) in the second task. That is, \( T_i(v_i) \) solves

\[
e^{-rT_i(v_i)} \kappa \left( \frac{r - e^{T_i(v_i)a}}{r - a} + \left( -1 + e^{rT_i(v_i)} \right) \bar{a} \right) \left( r - \bar{a} \right) = v_i, \quad (54)
\]

With this notation the principal’s objective becomes

\[
\int_0^\infty \sum_i \left( \sum_j W_i^2(v_{j,t}) - u_{i,t} - \sum_{j \neq i} v_{j,t} \right) a_{i,t}^1 e^{-y_i^1 - rt} \bar{p}^1.
\]

Let \( f_0(x, a, t) \) denote the integrand in the previous expression , where \( x \) is a vector that contains the state variables and \( a \) the control variables (under the rewriting of the problem \( v_i^l \) are control variables). Denote \( f_i(x, a, t) \) for \( i \in \{1, 2, 3\} \) for the differential equations given by equations (39) through (41) and \( f(x, a, t) = (f_i(x, a, t))_{i=1}^3 \). We will now establish existence of a solution to the principal’s problem by referencing Theorem 18 in page 400 of Seierstad and Sydsæter (1987). Define the set

\[
N(x, U, t) = \{(f_0(x, a, t) + v, f(x, a, t)) : v \leq 0, a \in U\},
\]

where \( U \) denotes the set of controls.

Let’s see that \( N(x, U, t) \) is convex. We will see that it is sufficient for the convexity of \( N(x, U, t) \)
that \( f_0 \) and \( f \) be concave in \( U \).

Consider the costly incentives case. First note that it is optimal for the principal to let the agent who succeeds experiment until the efficient threshold in the second task. Thus, it is without loss to consider the principal’s problem in which the control variables are the unsuccessful agent’s promised utility and the effort choices. The efficient threshold allocated to the successful agent in the second task depends on the experimentation threshold of the unsuccessful agent, and, therefore, so does his utility. Thus, we write \( v^j_{i,t}((v^i_{-i,t})) \). To show that \( f_0 \) is concave in \( v^j_{i,t} \) we show that \( W^2_i(v^i_{-i,t}) + W^2_i(v^j_{i,t}(v^i_{-i,t})) \) is concave in \( v^i_{-i,t} \).\(^{55}\) In fact, from equation (54) we have

\[
T'_i(v_i) = \frac{e^{\tilde{r}T_i(v_i)}}{\kappa \tilde{a} (e^{\tilde{a}T_i(v_i)} - 1) (1 - \tilde{p})},
\]

and

\[
T''_i(v_i) = -\frac{e^{2\tilde{r}T_i(v_i)}((\tilde{a} - r) e^{\tilde{a}T_i(v_i)} + r)}{\kappa^2 \tilde{a}^2 (e^{\tilde{a}T_i(v_i)} - 1)^3 (1 - \tilde{p})^2}.
\]

Using these expressions, the second derivative of \( W^2_i(v^i_{-i,t}) + W^2_i(v^j_{i,t}(v^i_{-i,t})) \) with respect to \( v^i_{-i,t} \) simplifies to

\[
\frac{(\kappa^2 - \pi^2) e^{(r-\tilde{a})T_i - \tilde{T_i}(\tilde{a} + r)} (\tilde{a} (e^{(\tilde{a} + r)T_i - \tilde{T_i}(\tilde{a} + r)} - e^0) - e^{0}) + r ( - 2 e^0 (\tilde{a} + r) - \tilde{T_i}(\tilde{a} + r) - \tilde{T_i}(\tilde{a} + r) + 3 e^0 (\tilde{a} + r) + 2 e^{0})}{\tilde{a} (\kappa^2) (\tilde{p} - 1)^2 (\tilde{a} + r) (e^{\tilde{a}T_i} - 1)^2}
\]

where to simplify notation we omitted the dependence of \( T_{-i} (v^j_{i,t}(v^i_{-i,t})) \) and \( T_i (v^j_{i,t}) \) on \( v^j_{i,t} \). If \( T_{-i} = T_i \) the previous expression is zero. Furthermore, the derivative of the term in parentheses in the numerator with respect to \( T_i \) is

\[
\tilde{r}(\tilde{a} + r)e^{\tilde{a}T_i}(3e^0 - 2e^{\tilde{a}T_i} - e^{\tilde{a}T_i}) \geq 0
\]

as we have \( T_i \geq T_{-i} \).\(^{56}\) Therefore, the second derivative of \( W^2_i(v^i_{-i,t}) + W^2_i(v^j_{i,t}(v^i_{-i,t})) \) is negative and \( f_0 \) is concave in \( v^i_{-i,t} \). \( f \) is trivially concave in the set of control because it is linear in them.

To see that \( N(x, U, t) \) is convex consider two controls \( a = (a_{i,t}, d_i, v^j_{i,t}, T_i(t))_{i,j} \) and \( \tilde{a} = (\tilde{a}_{i,t}, \tilde{d}_{i,t}, \tilde{v}^j_{i,t}, \tilde{T}_i(t))_{i,t} \), and real numbers \( v, v' \leq 0 \) we need to show that

\[
\beta (f_0(x, a, t) + v, f(x, a, t)) + (1 - \beta) (f_0(x, \tilde{a}, t) + v', f(x, \tilde{a}, t)) \geq 0
\]

is in \( N(x, U, t) \) for every \( \beta \). Since \( W^2_i(v_i) \) is concave, \( f_0 \) and \( f \) are concave in \( a \), thus, (55) is in

\(^{55}\)From the discussion given \( v^j_{i,t}, v^j_{j,t} \) is set as a function of \( v^i_{i,t} \) so that agent \( i \) works until the efficient belief.

\(^{56}\)\( T_i \geq T_{-i} \) at the optimum as is shown in section B.5
These computations are available upon request.

B.5 Costly first-task incentives

In the costly incentives case $\tilde{\gamma}_{i,t} = 0$ and $u_{i,t} > v_{i,t}(T_i^2(i,t))$ for every $t$. Let’s see that the principal sets $\tilde{\gamma}_{i,t} = 0$ and $a_{i,t} = \bar{a}$. Note first that $u_{i,t}$ in equation (35) increases in $\tilde{\gamma}_{i,t}$. Also, if $u_{i,t} > v_{i,t}(T_i^2(i,t))$ the principal has to give a strictly positive bonus to the agent in case of success. Setting $\tilde{\gamma}_{i,t} = 0$, if possible while maintaining $u_{i,t} > v_{i,t}(T_i^2(i,t))$, reduces the bonus and does not change the incentives to exert effort (because, as we have seen, when $\tilde{\gamma}_{i,t} = 0$ the necessary condition for effort is also sufficient). If $u_{i,t} > v_{i,t}(T_i^2(i,t))$ is violated when setting $\tilde{\gamma}_{i,t} = 0$ then the problem does not fall in the costly incentives case.

Thus, solving for $\lambda_{i,t}$ from equation (43) as $\tilde{\xi}_{i,t} = 0$ when $u_{i,t} > v_{i,t}(T_i^2(i,t))$ we obtain

$$
\lambda_{i,t} = (1 - \zeta_i)\bar{p}^1 e^{-\int_0^\infty (r + a_{i,t})ds} \left( 1 - e^{-\int_0^\infty a_{i,t}ds} \right).
$$

Replacing into equation (53) we obtain

$$
\left( (\zeta_i - 1)e^{-\int_0^\infty a_{i,t}ds} \left( e^{T_i(k,t)} - 1 \right) \right) \kappa^2 \left( 1 - \bar{p}^2 \right) + \bar{p}^2 \left( \pi^2 - \kappa^2 \right) e^{-2T_i(k,t)\bar{a}} - \kappa^2 (1 - \bar{p}^2)
$$

$$
- \bar{p}^2 \bar{a} \int_{T_i(k,t)}^{\infty} \left( \pi^2 - \kappa^2 \right) e^{-\bar{a}T_i(k,t) - \bar{a}s - r(s - T_i(k,t))} ds = 0.
$$

Which is the condition in equation (12).

Note that we must have $\zeta_i \leq 1$. When $W_{i,0} = \tilde{W}_{i,T_i} > 0$ then $\zeta_i = 1$ and $\lambda_{i,t} = 0$. The optimal thresholds are the efficient ones that the social planner implements.

Now, to see that $T_i^2(k, \tau)$ is decreasing in $\tau$ note that the term

$$
\left( \pi^2 - \kappa^2 \right) \bar{p}^2 e^{-T_i^2(k,\tau)\bar{a}} - \bar{p}^2 \bar{a} \int_{T_i^2(k,\tau)}^{\infty} \left( \pi^2 - \kappa^2 \right) e^{-\bar{a}\tau - \bar{a}s - r(s - T_i^2(k,\tau))} ds,
$$

For the other cases involving two symmetric agents with parameters that do not fall in the costly incentives case establishing the convexity of $N(x, U, t)$ requires more work. As in the costly incentives case, we need to show that a $f_0$ is concave to establish sufficiency. However, we cannot replace $v_{i,t}$ as a function of $v'_{i,t}$ which increases the dimensionality of the problem. We can show numerically that the Hessian of $f_0$ is concave in the promised utilities. These computations are available upon request.

When $\zeta_i > 1$ the equality in (57) cannot be satisfied since the first term is positive the second term is decreasing in $T_i(k,t)$ and zero at $T_i(k,t) = V_u/(2\bar{a})$. $V_u/(2\bar{a})$ is the maximum value $T_i(k,t)$ can take as it is not profitable for the principal to let agents experiment beyond the efficient stopping belief and $T_i(i,t) \geq T_i(k,t)$.
is decreasing in \( T_i^2(k, \tau) \). Its derivative with respect to \( T_i^2(k, \tau) \) is given by

\[
-2a \bar{p}^2 r (\pi^2 - \kappa^2) e^{-\frac{V_{a(t+\bar{r})}}{\bar{a}}} - 2a T_i^2(k, \tau) \left( e^{\frac{V_{a(t+\bar{r})}}{\bar{a}}} - e^{2T_i^2(k, \tau)(\bar{a}+r)} \right) \leq 0.
\]

where the inequality follows from \( T_k(k, \tau) = V_a/\bar{a} - T_i^2(k, \tau) \geq T_i^2(k, \tau) \). From equation (53), dividing by \( e^{-T_i^2(k, \tau)r} \) one can see that this implies that \( T_i^2(k, \tau) \) decreases in \( \tau \). Furthermore, whenever \( \xi_i < 1 \) (which includes the unconstrained case), \( T_i^2(k, \tau) \to 0 \) as \( t \to \infty \). In fact, the first term in (12) converges to \( \infty \) as \( t \to \infty \) if \( T_i^2(k, \tau) \) remains bounded above from zero. Thus, \( T_i^2(k, t) \) must converge to 0 as \( t \to \infty \) to maintain equality in (57).

From an analogous argument we can see that \( T_i^2(k, \tau) \) increases in \( \xi_i \).

Now, let’s see that the principal sets \( a_{i,t}^1 = \bar{a} \). Suppose \( a_{i,t}^1 < \bar{a} \) at some interval. Define

\[
\pi_{i,t} = \int_0^{T_i^2(i,t)} p_i^2(\pi^2 - w_{i,t} \tau)\bar{a}e^{-\int_0^{s} r_i^2(s)ds}ds d\tau.
\]

where \( w_{i,t} \) is agent \( -i \)’s bonus payment in task two if agent \( i \) succeeds in task 1 at time \( t \). \( \pi_{i,t} \) is the expected payoff that the principal receives in task two from agent \( -i \)’s experimentation after that history. Define

\[
\tilde{\pi}_{i,t} = \int_0^{T_i^2(i,t)} (p_i^2\pi^2 - \kappa^2)\bar{a}e^{-\int_0^{s} r_i^2(s)ds}ds d\tau.
\]

\( (\pi(T_i^2_1) + \tilde{\pi}(T_i^2)) \) is decreasing in \( t \). In fact, from the maximization of the Hamiltonian with respect to \( T_i^2_1(i,t) \) and \( T_i^2(i,t) \) we have

\[
\frac{d(\pi(T_i^2_1) + \tilde{\pi}(T_i^2))}{dt} = \frac{d(\pi(T_i^2_1) + \tilde{\pi}(T_i^2))}{dT_i^2(i,t)} \frac{\partial T_i^2(i,t)}{dt} = (1 - \xi_0)\int_0^{s} a_{i,t}^1 ds \frac{\partial v_i}{\partial T_i^2(i,t)} \frac{\partial T_i^2(i,t)}{dt} \bar{a} < 0,
\]

from \( \frac{\partial T_i^2(i,t)}{dt} < 0 \) and \( \frac{\partial v_i}{\partial T_i^2} = e^{-T_i^2(i,\tau \bar{r})} \left( -1 + e^{T_i^2(i,\tau \bar{r})} \right) \kappa^2 \bar{a}(1 - \bar{p}^2) > 0 \).

Consider the principal’s decision of shifting effort from time interval \([t, t+dt]\) to time interval \([t+dt, t+2dt]\). The principal’s payoff from agent \( i \)’s work in the first stage can be approximated as

\[
\Pi_{i,t} = (p_i^1 (\pi(T_{-i}) + \tilde{\pi}(T_i)) - \kappa^1 - (p_i^1 u_{i,t} - \kappa^1)) a_{i,t}^1 (1 - e^{-a_{i,t}^1 dt}) + e^{-(r+2a_{i,t} + a_{-i,t})dt} \Pi_{i,t+dt}.
\]

Replacing \( \Pi_{i,t+dt} \) recursively and replacing the exponentials by their second order Taylor ex-
pansion we obtain, from the derivations in section A.2,
\[
\frac{\partial}{\partial (dt)^2} \left( \frac{\partial \Pi_{i,t}}{\partial \epsilon} \right) = \frac{d}{dt} \left( \pi(T^2_{-i}) + \pi(T^2_{i}) \right) \rho_t^1 - (\pi(T^2_{-i}) + \pi(T^2_{i}) - \kappa^1) \rho_t^1 + \frac{\partial}{\partial (dt)^2} \left( \frac{\partial V_{i,t}}{\partial \epsilon} \right) < 0,
\]

where the inequality follows from equation (58) and from \( p_t^1 \left( \pi(T^2_{-i}) + \pi(T^2_{i}) \right) - \kappa^1 > 0 \), which is necessary for there to be a positive surplus from \( i \)'s experimentation.

Thus, the principal obtains a second order increase in her payoff from time \( t \) from shifting effort from interval \([t + dt, t + 2dt]\) to \([t, t + dt]\) in her payoff. By the argument in the proof of Proposition 2 from time zero to \( t \) the principal’s payoff is constant up to third order.

**Effects of increasing the outside option**  Let’s see that as the outside option increases the agents’ allocation provides more rents to each one of them in both tasks. Since the agents do not receive a positive lump-sum payment at time zero unless the allocations are the efficient ones (as shown in Section B.4) we know that the agents’ allocations must improve, i.e. provides more rents, on average as the outside option increases. From the discussion above we know that \( T^2_{i}(k,t) \) increases in \( \xi_t \). We now show that both \( T^1_{i} \) and \( T^2_{i}(k,t) \) increase as the outside option increases.

Define
\[
H^{RR}_{i,t} = \left( \pi(T^2(i,t)) - c(T^2(i,t)) - u_{i,t} - \sum_{j \neq i} v^i_{j,t}(T^2(i,t)) \right) a^1_{i,t} e^{-\gamma^1_{t} - r} \bar{p}^1 - \omega_i + \alpha_i \sum a^1_{i,t} + \lambda_{i,t} \left( (u_{i,t} - \kappa^1) (a^1_{i,t} - \rho^1) - \sum_{j \neq i} v^i_{j,t}(T^2(j,t)) a^1_{j,t} - \kappa^1 r e^{\gamma^1_{t} + \omega^1_0} \right) + \xi_{i,t} \left( (u_{i,t} - \kappa^1) e^{-\gamma^1_{t} \bar{p}^1} - (1 - \bar{p}^1) \kappa^1 \right) a^1_{i,t} e^{-rt} + \sum_{j \neq i} v^j_{i,t}(T^2(j,t)) a^1_{j,t} e^{-\gamma^1_{t} - r} \bar{p}^1.
\]

The transversality condition for time \( T^1_{i} \) is\(^{59}\)
\[
\tilde{H}_{T^1_{i}} = e^{-\gamma^1_{T^1_{i}} - r T^1_{i}} \left[ H^{RR}_{T^1_{i},i} + \tilde{\mu}_i \left( - \sum_{j \neq i} v^i_{j,t} a^1_{j,T^1_{i}} + G(y_{T^1_{i}, T^1_{i}}) e^{\gamma^1_{T^1_{i}} + r T^1_{i}} (a^1_{T^1_{i}+} + r) \right) \right] = 0,
\]

where \( a^1_{T^1_{i}+} = \lim_{t \to T^1_{i}+} a^1_{t} \).

\(^{59}\)This transversality condition corresponds to the transversality of the problem in which all wages and efforts of players other than \( i \) are given and the principal chooses the effort and wages of agent \( i \). The latter efforts and wages must be optimal given the former.
Thus,
\[
\frac{d\hat{H}_{T_i^1}}{d \zeta} = \frac{\partial H_{T_i^1}}{\partial \zeta} + \frac{\partial H_{T_i^1}}{\partial T_i^1} \cdot \frac{\partial T_i^1}{\partial \zeta} + \sum_{j,k} \frac{\partial H_{T_i^1}}{\partial T_j^2(k,T_i^1)} \cdot \frac{\partial T_j^2(k,T_i^1)}{\partial \zeta}.
\]

From the principal’s maximization \( \frac{\partial H_{T_i^1}}{\partial T_j^2(k,T_i^1)} = 0 \) for every \( j, k \).

Replacing the transversality conditions in (48) yields
\[
\frac{d\hat{H}_{T_i^1}}{d \zeta} = \left( \tilde{\rho} \left( e^{\rho \gamma_{i^1}^1 a_{i^1}^1 ds} - 1 \right) e^{\rho \gamma_{i^1}^1 + \rho \kappa_i a_{i^1}^1 + \rho \kappa_i^1 a_{i^1}^1 ds} \sum_{j \neq i} v_{\gamma_i^1} (T^2(j,t)) a_{i^1}^1 \left( a_{i^1}^1 - a_{i^1}^1 + e^{-\rho \gamma_{i^1}^1 a_{i^1}^1 ds} \right) \right)
+ \left( e^{\rho \gamma_{i^1}^1 a_{i^1}^1 ds} - 1 \right) G(y_{T_i^1}, T_i^1) e^{\rho \gamma_{i^1}^1 + \rho r T_i^1} (a_{i^1}^1 - a_{i^1}^1 + r) \tilde{\rho}^1 + G(y_{T_i^1}, T_i^1) e^{\rho \gamma_{i^1}^1 + \rho r T_i^1} a_{i^1}^1,
\]
which is positive.

Also, \( \hat{H}_t > 0 \) for \( t < T_i^1 \) and \( \hat{H}_t < 0 \) for \( t > T_i^1 \). Therefore, since \( T_i(k,t), k \neq i \), decreases in \( t \) we must have \( \frac{\partial H_{T_i^1}}{\partial T_i^1} > 0 \). Implying that \( \frac{\partial T_i^1}{\partial \zeta} \geq 0 \). This shows that \( \zeta \) increases as the outside option increases and that both \( T_i^1 \) and \( T_i(k,t) \) increase as the outside option increases.

**B.6 Cheap incentives in the first task**

Let \( \nu \) be the payoff that agents receive if they both work until threshold \( T^{2**} \) in task 2, i.e. \( \nu \equiv v_{i^1}(T^{2**}) \). The following condition guarantees that the primitives fall in the cheap first-task incentives case:

**Condition 1.** Suppose that either

1. \( \left( r \left( \kappa_i^1 e^{\rho \gamma_{i^1}^1 + \frac{2\rho \tilde{\alpha}}{\tilde{\alpha}} T^{2**} + \frac{2\rho \tilde{\alpha}}{\tilde{\alpha}} - \nu} + \tilde{\alpha} \kappa_i^1 \right) \right) = \left( r \left( \Pi^2(T^{2**}) - \nu \right) + \tilde{\alpha} \kappa_i^1 \right) < 0, \)

2. \( \left( r \left( \kappa_i^1 e^{\rho \gamma_{i^1}^1 + \frac{2\rho \tilde{\alpha}}{\tilde{\alpha}} - \nu} + \tilde{\alpha} \kappa_i^1 \right) \right) > 0 \) and \( T^1(T^{2**}) \leq \ln \left( \frac{a(2\nu - \kappa_i^1)}{2\tilde{\alpha} \kappa_i^1 e^{\rho \gamma_{i^1}^1 + \frac{2\rho \tilde{\alpha}}{\tilde{\alpha}} - \nu} + \tilde{\alpha} \kappa_i^1 + \kappa_i^1 e^{\rho \gamma_{i^1}^1 + \frac{2\rho \tilde{\alpha}}{\tilde{\alpha}} - \nu}} \right) \frac{1}{2\tilde{\alpha} + r}, \) or

3. \( T^1(T^{2**}) \leq \ln \left( \left( \frac{\nu}{\tilde{\alpha}^1} - \frac{1}{2} \right) e^{-\rho \gamma_{i^1}^1 a_{i^1}^1 \left( \frac{\nu}{\tilde{\alpha}^1} - \frac{\tilde{\alpha}^1}{r} - 1 \right) \frac{\tilde{\alpha}^1}{2\tilde{\alpha} + r}} \right) \frac{1}{2\tilde{\alpha} + r} \) holds.

where \( T^{2**} = (T^{2**}, T^{2**}) \)

We now show that under Condition 1 the agents exert maximum effort until time \( T^1 \) if they all receive the same time independent reward, \( \nu \), when an agent succeeds in completing the first task. From equations (53) and (52), if \( \bar{r}_{i,t} > 0 \) the second task assignments do not depend on the history. Thus, we have \( v_{i,t}^j = v_{i,t}^j \equiv \nu \) for all \( i, j \) and \( t \). The second task payoff \( \nu \) increases in
the multiplier $\zeta_i$ and $\zeta_i$ must increase in the outside option.\(^{60}\) Thus, players with a higher outside option receive better allocations in the second task.

The multiplier, $\tilde{\gamma}_{i,t}$, that solves equation (33) when $w^i_{t_1} = 0$, $v_{i,t}^i = v$ and $v^j_{i,t} = v$ is strictly positive for $t \leq \bar{T}^1$ if and only if assumption 1 holds. In fact, (33) becomes

$$\tilde{\gamma}_{i,t} \equiv r \tilde{\gamma}_{i,t} - e^{-\gamma t_i} (v - \kappa^1) r + e^{-\gamma t_i} \kappa^1 \tilde{a}_{i-1,t} + \kappa^1 r e^{\gamma t_i}. $$

The transversality condition in equation (48) is $\tilde{\gamma}_{i,\bar{T}^1} = (v - \kappa^1) e^{-\gamma t_i} - \kappa^1 e^{\gamma t_i}$. The previous differential equation can be solved in close form and yields a unique function $\tilde{\gamma}_{i,t}$. Integrating both sides of the previous equation we obtain

$$\tilde{\gamma}_{i,t} e^{-rt} - \tilde{\gamma}_{i,T^1} e^{-rT^1} = \int_t^{\bar{T}^1} e^{-rs} \left( e^{-\gamma t_i} (v - \kappa) r - e^{-\gamma t_i} \kappa^1 \tilde{a}_{i-1,t} - \kappa^1 r e^{\gamma t_i} \right) ds$$

which yields

$$\tilde{\gamma}_{i,t} e^{-rt} = \frac{\dot{a}(2v - \kappa^1) e^{-T^1(2\kappa r + \gamma) - (\gamma t_i - \kappa^1 \tilde{a}_{i-1,t} + \gamma t_i r e^{\gamma t_i})}}{2\kappa r + \gamma} + \frac{e^{-t(2\kappa r + \gamma)} (rv - \kappa^1 (\tilde{a} + r) - \kappa^1 r e^{\gamma t_i})}{2\kappa r + \gamma} - \kappa^1 r e^{\gamma t_i} - r.$$  

Note that $\tilde{\gamma}_{i,t} e^{-rt}$ is decreasing with respect to $\bar{T}^1$. The derivative of $\tilde{\gamma}_{i,t} e^{-rt}$ with respect to $t$ is given by $e^t \left( -(2\kappa r + \gamma) \right) \left( r (\kappa^1 e^{2\kappa r + \gamma t_i} + \kappa^1 - v) + \kappa^1 \right)$. We now characterize the set of primitives that imply $\tilde{\gamma}_{i,t} > 0$ for all $t \in [0, \bar{T}^1]$.

**Case 1:** $(r \left( \kappa^1 e^{\gamma t_i + 2\kappa r + \gamma t_i} + \kappa^1 - v \right) + \kappa^1) < 0$.

In this case $\tilde{\gamma}_{i,t} e^{-rt}$ is decreasing and reaches its lowest value at $t = \bar{T}^1$. By the transversality condition we have $\tilde{\gamma}_{i,T^1} e^{-rT^1} > 0$, and therefore $\tilde{\gamma}_{i,t} > 0$ for all $t \leq \bar{T}^1$.

**Case 2:** $(r \left( \kappa^1 e^{\gamma t_i + \kappa^1 - v} + \kappa^1 \right) + \kappa^1) > 0$.

In this case $\tilde{\gamma}_{i,t} e^{-rt}$ is increasing and reaches its lowest value at $t = 0$. The condition for $\tilde{\gamma}_{i,0} \geq 0$ is

$$\bar{T}^1 \leq \ln \left( \frac{\dot{a}(2v - \kappa^1)}{2\kappa r + \gamma} \right) \frac{1}{2\kappa r + \gamma}.$$  

**Case 3:** $(r \left( \kappa^1 e^{\gamma t_i + 2\kappa r + \gamma t_i} + \kappa^1 - v \right) + \kappa^1) = 0$ for some $\tau \in [0, \bar{T}^1]$.

In this case $\tilde{\gamma}_{i,t} e^{-rt}$ is decreasing in $t \in [0, \tau)$ and increasing in $t \in (\tau, \bar{T}^1]$. It reaches its lowest value at $\tau = \left( \ln \left( \frac{-\dot{a}(2v - \kappa^1 \tilde{a}_{i-1,t} + \gamma t_i r e^{\gamma t_i})}{2\kappa r + \gamma} \right) - x_0 \right) \frac{1}{2\kappa r}$. The condition for $\tilde{\gamma}_{i,t} \geq 0$ is that

$$\bar{T}^1 \leq \ln \left( \left( \frac{v}{\kappa^1} - \frac{1}{2} \right) e^{-\gamma t_i - x_0} \left( \frac{v}{\kappa^1} - \frac{\kappa^1}{r} - 1 \right) \frac{\kappa^1}{2\kappa r + \gamma} \right) \frac{1}{2\kappa r + \gamma}.$$  

\(^{60}\) Since the agents are symmetric, the same multiplier equates the agent’s payoff to the outside option $\bar{O}$.  

\[69\]
To see that the principal prefers that the agents exert full effort until the efficient threshold, note that since \( \pi(T_i^2) \) and \( \tilde{\pi}(T_i^2) \) are constant in \( t \) we obtain as before
\[
\frac{\partial}{\partial (dt)^2} \left( \frac{\partial \Pi_{i,t}}{\partial \epsilon} \right) = -\left( a_{i,\epsilon} = r + (\pi(T_{-i}) + \tilde{\pi}(T_i)) - \kappa \right) p_i^1 + r \kappa e^{\alpha t} p_i^1 + \frac{\partial}{\partial (dt)^2} \left( \frac{\partial V_{i,t}}{\partial \epsilon} \right) < 0. \tag{60}
\]
Thus, the principal does not want to delay the agents’ work.

### B.7 Intermediate costs case: Proof of Proposition 8

We need to show that there is a time interval in which the payoff of agent \( i, u_{i,t} \), satisfies the differential equation for \( u_{i,t}^{\min} \) in equation (9) and, at the same time \( u_{i,t} = v_{i,t}^{\prime} \). That is, the agent does not receive a bonuses for the first-task discovery. Note that in the intermediate incentives cost case \( u_{i,t} \) might not satisfy the equation for \( u_{i,t}^{\min} \) for every time \( t \). That is, \( \gamma_{i,t} \) in equation (33) must be zero at a set of times (which is what we need to show in the present proof) and may be strictly positive at other times.

If \( \gamma_{i,t} > 0 \) for all \( t \) and \( u_{i,t} = v_{i,t}^{\prime} \), then from the analysis in B.4 and equation (52), we must have \( \gamma_{i,t} = 0 \) for all \( t \) and, therefore, \( T_i^{2}(i,t) = T_i^{2}(i,t) = T^{2*} \), where \( T^{2*} \) is the principal’s optimal experimentation threshold in a one-task project with the parameters of task 2. In such case, the parameters of the problem fall in the cheap incentives case.

On the other hand, if \( \gamma_{i,t} = 0 \) and \( u_{i,t} > v_{i,t}^{\prime} \) for all \( t \) then the parameters fall in the costly incentives case, as in section B.5.

Therefore, in the intermediate costs case we must have that either (1) \( \gamma_{i,t} = 0 \) and \( u_{i,t} = v_{i,t}^{\prime} \) for all \( t \) or (2) that there are sets of times \( \mathcal{A}_1, \mathcal{A}_2 \neq \emptyset \) such that \( \gamma_{i,t'} > 0 \) and \( u_{i,t'} = v_{i,t'}^{\prime} \) for every \( t' \in \mathcal{A}_1 \) and \( \gamma_{i,t''} = 0 \) and \( u_{i,t''} > v_{i,t''}^{\prime} \) for every \( t'' \in \mathcal{A}_2 \). If (1) holds we obtain the desired conclusion.

Let’s see that (2) implies that there is an interval \([t_0,t_1]\) in which \( \gamma_{i,t} = 0 \) and \( u_{i,t} = v_{i,t}^{\prime} \) for \( t \in [t_0,t_1] \). Suppose not. As \( \gamma_{i,t} \) is continuous there is an interval \([t_0',t_1']\) such that \([t_0',t_1'] \subseteq \mathcal{A}_1 \) and \( t_1' \in \mathcal{A}_2 \). From equation (51), \( T_i^{2}(i,t') \) is the efficient threshold. As \( \tilde{\gamma}_{i,t} > 0 \) for \( t \in [t_0',t_1'] \), and \( \gamma_{i,t} \) is continuous, \( \tilde{\gamma}_{i,t} > 0 \) for every \( t \in [t_0',t_1'] \). Therefore, from equation (52), \( T_i^{2}(i,t) \) is bounded away from the efficient threshold in \([t_0',t_1']\). But since \( T_i^{2}(i,t') \) is the efficient threshold, \( u_{i,t} \) must be discontinuous, which is a contradiction.
B.8 Conditions for an asymmetric contract

Let $T^S$ solve the following equation

$$v^j_{i,T} \left( T_{-iT}(i,T^S) \right) e^{T^S \tilde{a}} + \kappa^1 e^{3T^S \tilde{a} + x_0} + \kappa^1 + c \left( T_i^2(i,T^S), T_{-iT}(i,T^S) \right) - \pi \left( T_i^2(i,T^S), T_{-iT}(i,T^S) \right) = 0.$$  
(61)

This equation corresponds to the first order condition for the first task optimal symmetric experimentation thresholds.

**Proposition 13.** A sufficient condition for the first task contract to be asymmetric is

$$- \kappa^1 e^{T \tilde{a} + x_0} \left( - \tilde{a} e^{T(\tilde{a}+r) + 2T^S \tilde{a}} + (\hat{a} + r) e^{T^S(\tilde{a}+r) + 2T \tilde{a}} + r \left( - e^{T^S(3\tilde{a}+r)} \right) \right)$$

$$+ v^j_{i,T} \left( T_i^2(T) \right) \left( e^{T(2\tilde{a}+r)} - e^{\tilde{a}(T+T^S)+rT^S} \right) \tilde{a} < 0,$$

for $T \in [T^S - \varepsilon, T^S]$ for some $\varepsilon$.

Suppose the principal sets first-task thresholds $T$ and $T^S$, with $T < T^S$, for agents $i$ and $-i$, respectively. The expression in (62) is the first-order condition of the principal’s payoff with respect to $T$ (the first stopping time), replacing $\pi(\cdot) - c(\cdot)$ from the first order condition for $T^S$. If it is negative for $T \in [T^S - \varepsilon, T^S]$ for some $\varepsilon$, the principal’s payoff increases as $T$ decreases.

The second term negative in equation (62), while the first term is positive. For a given threshold $T^S$ in equation (61) one can decrease $x_0$ and decrease $\pi_1$ (which is an additive variable in $\pi(\cdot)$), simultaneously, so as maintain the equality. Thus, there are small enough $x_0$ and $\pi_1$ such that the optimal contract is asymmetric.

B.9 Proof of Proposition 9

From the arguments in the one-task case, the principal prefers to disclose the second-task breakthroughs as soon as they occur.

In the cheap incentives case the principal discloses breakthroughs right away. Since she does not need to provide rewards for the first-task breakthroughs, there are no possible cost savings from delay.

Suppose the parameters fall in the costly incentives case. We saw that delaying disclosure is not beneficial to reduce procrastination rents. Thus, the potential benefit of delaying disclosures is the reduction in public-good rents. Let $d_{j,s}$ be the rate at which agent $j$’s breakthroughs are disclosed from an ex-ante perspective. The cost of inducing effort by agent $i$ is given by

$$u^{\text{min}}_{i,t} = w_{i,t}(1) + \sum_{j \neq i} \int_t^\infty e^{-\int_t^s (r + \sum_{k \neq i} d_{k,s}) ds} \bar{d}_{j,\tau} v^j_{i,\tau} d\tau.$$
The principal’s expected profit arising from agent $i$’s experimentation in histories in which agent $j$’s discovery is disclosed given the disclosure policy, is given by

$$
\int_{0}^{\infty} \left( \pi_i(\tilde{T}_i^2(j,t)) - c_i(\tilde{T}_i^2(j,t)) - v_{i,t} (\tilde{T}_i^2(j,t)) \right) \tilde{d}_{j,t} e^{-\int_{0}^{\tilde{T}_i} (a_{i,s} + \tilde{\Sigma}_{k,s} \tilde{d}_{k,s}) ds - rt} dt,
$$

where $\tilde{T}_i(j,t) = (\tilde{T}_i^2(j,t), \ldots, \tilde{T}_i^m(j,t))$ are the experimentation thresholds (measured from time $t$) if $j$’s discovery is disclosed at time $t$,

$$
\pi_i(\tilde{T}_i^2(j,t)) = \int_{0}^{\tilde{T}_i^2(j,t)} p_{i} \pi^2 a_i e^{-\int_{0}^{\tilde{\tau}} (p_i a_i + r) ds} d\tau + \pi^1
$$

and

$$
c_i(\tilde{T}_i^2(k, \tilde{\tau})) = \sum_{j} \int_{0}^{\tilde{T}_i^2(j,t)} k_i \tilde{a}_i e^{-\int_{0}^{\tilde{\tau}} (p_i a_i + r) ds} d\tau.
$$

The expectation of the public-good rents paid to agent $i$ that are incurred due to the disclosures of $j$’s discoveries is given by

$$
\int_{0}^{T_i^1} \left( \int_{t}^{\tau} e^{-\int_{s}^{\tilde{T}_i} (r + \tilde{\Sigma}_{k,s} \tilde{d}_{k,s}) ds} d\tilde{d}_{j,t,v_{i,t}} d\tilde{\tau} \right) a_{i,j} e^{-\int_{0}^{\tilde{T}_i} (a_{i,s} + \tilde{\Sigma}_{k,s} \tilde{d}_{k,s}) ds} dt
$$

$$
= \int_{0}^{T_i^1} \left( \int_{0}^{\tau} a_{i,s} e^{-\int_{0}^{\tilde{T}_i} (a_{i,s} + \tilde{\Sigma}_{k,s} \tilde{d}_{k,s}) ds} d\tilde{d}_{j,t,v_{i,t}} d\tilde{\tau} \right) e^{-\int_{0}^{\tilde{T}_i} (r + \tilde{\Sigma}_{k,s} \tilde{d}_{k,s}) ds} d\tilde{d}_{j,t,v_{i,t}} d\tau.
$$

The expected profit the principal obtains from $j$’s experimentation over histories in which $j$’s discovery is disclosed is

$$
\int_{0}^{\infty} \left( \pi_j(\tilde{T}_j^2(j,t)) - c_j(\tilde{T}_j^2(j,t)) \right) \tilde{d}_{j,t} e^{-\int_{0}^{\tilde{T}_j} (a_{j,s} + \tilde{\Sigma}_{k,s} \tilde{d}_{k,s}) ds - rt} dt.
$$

Summing up the terms that depend on $\tilde{T}_i(j,t)$ we obtain

$$
\int_{0}^{\infty} \left( \pi_i(\tilde{T}_i^2(j,t)) - c_i(\tilde{T}_i^2(j,t)) + \pi_j(\tilde{T}_j^2(j,t)) - c_j(\tilde{T}_j^2(j,t)) - e^{-\int_{0}^{\tilde{T}_i} a_{i,s} ds} v_{i,t} (\tilde{T}_i^2(j,t)) \right) \tilde{d}_{j,t} e^{-\int_{0}^{\tilde{T}_j} (a_{j,s} + \tilde{\Sigma}_{k,s} \tilde{d}_{k,s}) ds - rt} dt
$$

The optimal contract must maximize this expression with respect to $\tilde{T}_i^2(j,t)$. The first order condition for $\tilde{T}_i^2(j,t)$ coincides with the first order condition for $T_i^2(j,t)$ in the costly incentives case with automatic disclosure, and decreases in $t$. Thus, by the same argument as in the costly incentives case the principal obtains a second order gain from shifting an $\epsilon$ rate of disclosure in an interval of time of length $dt$ to the previous length $dt$ time interval, at a cost that is at most of third order.