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Goals, Constraints, and Public Assignment: A Field Study of the UEFA Champions League
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GOALS, CONSTRAINTS, AND PUBLIC ASSIGNMENT
A FIELD STUDY OF THE UEFA CHAMPIONS LEAGUE

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ABSTRACT. We analyze a dynamic matching mechanism developed for the UEFA Champions League, the largest and most-watched football club competition worldwide. First, we theoretically characterize the assignment rule developed by UEFA by solving a complex constrained assignment problem with a publicly verifiable draw. Then, using a structural model of the assignments and data from the UEFA Champions League 2004 and 2018 seasons we show that the constraints cause quantitatively large spillovers to unconstrained teams. Nevertheless, we conclude that the UEFA draw is close to a constrained-best in terms of fairness. Moreover, we find that it is feasible to substantially reduce the distortions by only marginally slacking the constraints.

1. INTRODUCTION

When designing an institution economists typically first prioritize the theoretic properties, sometimes to the extent that the mechanics of the solution can become too complex for participants to understand or follow. However, in some high-stake public settings, the focus is instead placed on transparency and simplicity, potentially at a cost to the mechanisms’ theoretic properties. In the present paper, we examine a randomized matching procedure designed to solve a complex assignment problem in a transparent and comprehensible manner. The designed draw assembles football team pairs in an environment with huge public interest: the Union of European Football Association’s (UEFA) Champions League (UCL), the most prestigious club-competition in football. While the randomization’s transparent procedure helps maintain the integrity of the draw under a series of non-trivial matching constraints, through a recent market-design tool we show that to all practical extents the chosen randomization is a constrained-best.

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The UCL has been one of the most successful pan-European ventures over the past half-century, and certainly the one with the most enthusiasm from the public. The tournament brings together football clubs from across the continent (and beyond) that normally play within country-level associations. Selection into the competition is limited to the top-performing clubs from each nation, where countries with deeper footballing history are allotted more places. A series of initial qualifying rounds whittle the number of participating teams down to 32 group-stage participants. From there, half of the clubs advance to a knockout stage that begins with the Round of 16 (R16), followed by four quarterfinals, two semifinals, and a final that determines a European champion. Outside of the World Cup, the UCL final game is one of the most-watched global sporting events, eclipsing even the viewership of the Superbowl in the United States.

Because the UCL has been put under a magnifying glass by both the public and partisan fans, UEFA has a clear interest in creating impartial, meritocratic, and publicly verifiable rules for the tournament. While a fair, in-public assignment rule would be simple to develop in many situations—for instance, matching teams through a standard urn draw without replacement—the tournament’s problem is complicated due to three constraints imposed on match pairs: (i) Each pairing must be between a group winner and a group runner-up (the bipartite constraint). (ii) Teams that played one another in the prior group stage cannot be matched (the group constraint). (iii) Teams from the same national association cannot be matched (the association constraint).

In order to address the need for a randomization over constrained assignments with a public and transparent draw, UEFA developed a novel procedure. While respecting the constraints, the UEFA randomization assembles each R16 team pair through a dynamic public draw of balls from an urn, where the urns composition dynamically adapts to the assignment constraints and the current state of the draw. Our paper sets out to understand and analyze this field-proven mechanism, using the main tools of market design: theory, estimation, and computer simulation (Roth, 2002).

To begin our study we first formalize the natural to follow but combinatorically complex draw procedure used by UEFA. Next, we shift to a quantitative analysis of the constraint effects, showing that exclusions of same-nation match-ups (the association constraint) create large distortions in teams’ progression probabilities within the tournament. For example, we find that the constraints affect the chances of advancing to the tournament’s semifinals by up to 20 percent, with an effect on expected tournament winnings of up to one million euro. Moreover, we demonstrate that even though two teams $A$ and $B$ might not be directly
excluded from a match with team \( C \), there can be substantial spillovers caused by others’
constraints, forcing asymmetries in the likelihoods of the \( AC \) and \( BC \) matches.

Given the spillovers from the constraints under the current procedure, a natural question
is whether an alternative constrained-matching procedure exists that results in a fairer ran-
domization. Using an objective function tailored to the idea of equal treatment of equals, we
apply a recent theorem from the market-design literature (Budish et al., 2013) to address
this normative question. The theorem generalizes the Birkhoff–von–Neumann decomposi-
tion to allow for matching constraints, and can be applied to our setting as the UEFA
constraints satisfy a separability condition across the two sides of the match. By relaxing
the combinatorically complex problem of looking for randomizations over constrained as-
signments, we focus (without loss of generality) on the more-tractable problem of finding
expected assignments satisfying the constraints. The conclusion from our analysis of the
UEFA problem across the past fifteen years shows that even though marginal improvements
are possible over the current procedure, the tournament’s randomization is qualitatively close
to a constrained-best.

While our search for better constrained randomizations suggests only minimal scope for
improvement—with large potential complexity costs from forgoing UEFA’s simple-to-follow
draw procedure—a related question is the extent to which much-fairer outcomes are possible
from slacking the constraints. As a constructive exercise, we end the paper by showing that
a small relaxation of the association constraint can go a long way towards reducing the dis-
tortions. Moreover, slacking the constraints can be obtained with only minimal adjustment
to the current matching rule retaining its transparency.

In summary, our paper documents a solution to a constrained matching procedure with three
desirable properties: (i) the dynamic procedure is simple-to-follow, despite the underlying
combinatoric complexity; (ii) the randomization is carried out publicly with fair urn draws,
helping to dispel doubts about the designer cherry-picking realizations; and (iii) it produces
expected assignments that are close to optimal (from a fairness point of view). General-
izations of the mechanism might be useful in many constrained market-design applications,
where openness and fairness in the randomization are paramount. For example, assignment
rules could be modified to fairly and publicly assign students to schools (Abdulkadiroğlu et
al., 2005; Abdulkadiroğlu and Sönmez, 2003; Ehlers et al., 2014; Pathak and Sönmez, 2013)
and class schedules (Krauss et al., 2013) in small markets. Similarly, the procedure could be
adapted to openly randomize donor/recipient pairs in kidney-exchange chains (Roth et al.,
2004, 2007, 2005; Ünver, 2010; Caragiannis et al., 2015). While many matching procedures
with desirable theoretical properties have been developed, algorithms can seem opaque to
participants. The application we study offers an alternative that instead prioritizes transparency of the draw, while still embedding a series of complex match constraints.

In terms of organization, Section 2 provides a brief review of the related literature. In Section 3, we describe the background of the UCL and discuss the constraints imposed on the R16 matching. In Section 4, we formalize the UEFA mechanism and some theoretic properties. In Section 5, we provide a description of the data used in estimating our goal-outcome model, and then present our quantitative results. Finally, Section 6 concludes.\textsuperscript{1}

2. Literature review

Our paper contributes to two main strands of economic literature: market design and tournaments. While there is large theoretic literature on the incentive effects of tournaments (see Prendergast, 1999) our paper is more closely related to a growing body of applied work exploiting sports-tournament outcomes as naturally occurring experiments. In recent years, using sports data from football through cricket to golf, the applied literature has provided evidence both for consistency with theory (Walker and Wooders, 2001; Chiappori et al., 2002; Palacios-Huerta, 2003) and behavioral biases (Bhaskar, 2008; Apesteguia and Palacios-Huerta, 2010; Pope and Schweitzer, 2011; Foellmi et al., 2016).

Where the literature on sports tournaments has been centered around various positive aspects of players’ behavior, our paper instead emphasizes normative features. In this sense, our work is more closely related to the market design literature, and a handful of applied papers examining well-structured environments. For instance, Fréchette et al. (2007) demonstrate the problem of inefficient unraveling in a decentralized market using data on US college football; Baccara et al. (2012) investigate spillovers across participants and their subsequent inefficiency using data on faculty office assignments; and Budish and Cantillon (2012) study the superiority of a manipulable mechanism to the strategy-proof mechanism using data from the assignment of courses in a business school. In each of these cases, conclusions are linked to applications of market design and matching theory. Similarly, our paper attempts to engineer an optimal randomization by combining theory, estimation, and simulation (see Roth, 2002).

One of the main market-design tools we employ is the core theorem in Budish et al. (2013) that facilitates the problem of seeking optimal matching randomizations, and enables us to show near-optimality of the existing UEFA assignment rule. This paper builds upon

\textsuperscript{1}An Appendix presents proofs of propositions together with additional (theoretical and empirical) results for interested readers. Full data, programs, and the Online Appendix are available at https://sites.google.com/site/martaboczon.
Hylland and Zeckhauser (1979) and Bogomolnaia and Moulin (2001) by showing sufficiency for expected assignment matrices under separable constraints known as a biheirarchy. To our knowledge our paper is one of the first ones to apply these market-design tools to normatively assess a field randomization. While our tournament setting is of standalone interest,\(^2\) the overall methodology has clear parallels to ongoing academic debates on school choice and the implementation of affirmative-action constraints (for example, Dur et al., 2016). Our paper introduces a novel design consideration that contributes to a literature focused on fairness, efficiency and strategy proofness in school choice (see Abdulkadiroğlu and Sönmez, 2003, and references thereof). Instead of designing around a concern over non-truthful behavior of mechanism participants, we instead focus on design around the potential for deceptive behavior on the part of principals, and the potential for opaque features in the randomization to be manipulated. This leads to an additional and nontrivial design challenge: how to construct a randomization procedure subject to complex matching constraints that is both fair to participants and transparent to outside observers.

3. Application Background

The UCL is the most prestigious worldwide club competition in football. Its importance within Europe is similar to that of the Superbowl in the United States, though with stronger global viewership figures (details sub.). The tournament is played annually between late June and the end of May by the top-division teams from 57 national associations across Europe, featuring most of the sports’ star players.

In the 2017 tournament, 6.8 million spectators attended UCL matches and more than 65,000 attended the final game alone.\(^3\) In addition to the in-stadium audience, the tournament has vast media exposure. For example, the UCL final game has become the most-watched annual sporting event in the world, where the 2015 final had an estimated global reach of 400 million viewers across 200 countries, and a projected live audience of 180 million. For comparison, the average Super Bowl viewership in the United States was equal to 103 million in 2018, 111 million in 2017, and 112 million in 2016.\(^4\)


\(^3\)Since each UCL season spans across two calendar years, for clarity purposes we refer to a particular season by the year of its final game (so 2018 for the 2017-18 season).

\(^4\)Furthermore, the UCL proves a massive success on social media. In the 2017 season, the UCL official Facebook page became the worlds’ most followed for a sporting competition with 63 million fans, 300 million video views, and 98 million interactions over the final game alone.
In financial terms, UEFA’s main revenue stream comes from selling broadcasting and commercial rights to UEFA’s three major club competitions: the UCL, the UEFA Europa League, and the UEFA Super Cup. In the 2017 season, UEFA generated a total revenue of 2.84 billion euro, where 75 percent of that amount was paid out to clubs and associations participating in UEFA competitions.\(^5\) For instance, in the 2017 season those clubs that advanced to the UCL group stage were awarded 12.7 million euro, while those that reached the R16 garnered a further 6 million (with group winners receiving a bonus of 1.5 million). Beyond the R16, quarter-finalists won an additional 6.5 million euro, semi-finalists 7.5 million, and finalists 11 million (with the winner receiving a bonus of 4.5 million).\(^6\)

Introduced in 1955 as a European Champion Club’s Cup open only to the national champions from each association, the tournament has evolved over the years to admit multiple entrants from the same national association (at most five).\(^7\) The last major change to the tournament’s design took place in the 2004 season. As such, in our empirical analysis we focus our attention on the seasons between 2004 and 2018 (the latest completed season at the time of writing).

Since the 2004 season, the UCL has consisted of a number of pre-tournament qualifying rounds followed by a group and then a knockout stage, similar in format to the World Cup, but played simultaneously with the European club seasons.\(^8\) In the group stage, 32 teams are divided into eight groups of four.\(^9\) Beginning in September each team plays the other three group members twice (once at home, once away). At the end of the group stage in December, the two lowest-performing teams in each group are eliminated, where the group winner and runner-up advance to the knockout stage. The knockout stage (except for the final game) follows a two-legged format, in which each team plays one leg at home. Teams that score more goals over the two legs advance to the next round, where the remaining teams are eliminated.\(^10\)

\(^5\)The first-order beneficiaries are clubs participating in the group stage onward. The remaining UEFA revenue is distributed among second- and third-order beneficiaries, clubs participating in the qualifying rounds and non-participating clubs, respectively. The solidarity payments made to the latter teams are distributed via their national associations and allocated, for the most part, to youth training programs.


\(^7\)For more details regarding the format changes, see Table 4 in the Appendix.


\(^9\)Prior to the 2015 season, teams were entirely seeded based on UEFA club coefficients, with the titleholder being automatically placed in Pot 1. Starting from the 2016 season, the titleholder and the champions of the top seven associations based on UEFA country rankings are being placed in Pot 1. The remaining teams are seeded to Pots 2-4 based on their UEFA club coefficients. The eight groups are formed by making sequential draws from the four pots with a restriction that teams from the same association cannot be drawn against each other, enforced in a similar way to the R16 match we detail in the paper.

\(^10\)Technically, the scoring rule is lexicographic over total goals, and goals away from home. A draw on both results in extra time, and then a penalty shootout until a winner is determined.
Our focus is on the assignment problem of matching the 16 teams at the beginning of the knockout phase into eight mutually disjoint pairs.\textsuperscript{11,12} While the bipartite and group constraints impose symmetric restrictions, the association constraint creates a strong asymmetry that affects the entire matching.\textsuperscript{13} If the problem consisted simply of matching two equal-sized sets of teams, two urns (one for group winners, one for runners-up) could be used to assemble the matching through a sequential draw without replacement. However, the presence of the constraints implies that a draw cannot be conducted this way for two reasons. The first is direct and easy to address. At all points during the draw, the set of eligible partners cannot contain those excluded by either the association or the group constraint. The second is indirect and requires more-complicated combinatoric inferences. Creating a match with a seemingly valid partner can force an excluded match at a later point in the draw, and as such must be excluded. To illustrate, consider a dynamic draw from two pots, one with teams $A$, $B$ and $C$; one with teams $d$, $e$ and $f$. Suppose that the matches $Ad$ and $Be$ are excluded. An initial draw from the first pot selects team $A$. The subsequent draw must directly exclude $d$. Assume $f$ is selected. In the second round, $C$ is chosen from the first pot. Although $C$ has no directly excluded partners, the constraints indirectly imply that $C$ cannot match with $d$. The reasoning is that if $Cd$ is formed, $B$ will have no valid partners in the third and final round. As such, given the initial $Af$ draw, $(Af, Ce, Bd)$ is the unique matching satisfying the constraints.

Although this logic is easy to follow when matching three teams to three teams, with eight teams on each side and more constraints, the combinatorics become involved. While matchings could be easily formed via fully computerized draws, UEFA instead opted to make much of the randomization transparent through an urn procedure. The dynamic draw procedure UEFA chose to randomize the R16 tournament matching works as follows: (i) eight blue balls representing eight runners-up are placed in the first urn and one ball is drawn without replacement; (ii) a computerized algorithm determines the set of group winners that can possibly match with the drawn runner-up given the constraints and the state of the current draw; (iii) white balls representing possible match partners determined in the previous step are placed into a second urn and one ball is drawn without replacement; (iv) a pairing of the two drawn teams (one winner and one runner-up) is added to the aggregate R16 matching. This procedure repeats until eight matches are formed. In what follows, we refer to the

\textsuperscript{11}Note that the quarter- and semifinal draws are free from seeding and association protection, and as such, are conducted in a standard fashion by drawing balls from an urn without replacement.

\textsuperscript{12}A political constraint excludes Russian teams from being drawn against Ukrainian teams. In what follows, we re-interpret this restriction as an association constraint.

\textsuperscript{13}In the presence of just the bipartite and group constraints, each non-excluded matching is equally likely.
above algorithm as the constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw, where $\mathcal{W}$ and $\mathcal{R}$ stand for the sets of group winners and runners-up, respectively.

Key features of the UEFA procedure are its simplicity and openness. Representatives from each R16 team attend the draw ceremony in order to verify the draw procedure in person. Moreover, the event draws substantial attention from the media and public. The 20-minute draw ceremony is streamed live by UEFA over the Internet and broadcast by many national media companies. Examining the viewership figures for the Internet stream, the most recent UCL R16 draw attracted 813,000 viewers on UEFA.tv alone.\(^{14}\) While this figure may be small relative to game-day TV audiences, it is a huge viewership when considering centralized assignment.

In the absence of the association constraint, the tournament has 14,833 possible matchings for the R16 teams, where each same-nation exclusion significantly reduces the number of valid assignments. Across the 15 most-recent seasons, the number of valid assignments ranges from 2,988 to 9,200. We graph the relationship between the number of possible matchings and the number of exclusions implied by the association constraint in Figure 1. While the number of valid assignments is not purely a function of the number of exclusions (it depends on their arrangement too) the relationship in question can be approximated by a linear function that decreases by 1,400 matchings for each same-nation exclusion.\(^{15}\)

4. Theory for the Current Matching Procedure

We now turn to the theory for the current procedure, where we describe a generalized version of the dynamic mechanism used by UEFA to draw assignments satisfying the constraints. After characterizing the effective lottery over matchings induced by the dynamic draw, we show that the randomization in question is distinct from simpler static implementations, as well as from a number of dynamic variants. Finally, we demonstrate that the constraints enforced by UEFA allow us to apply the main result in Budish et al. (2013) in order to analyze the extent to which the UEFA mechanism might be improved upon.

4.1. Constrained Dynamic Draw. Let $\mathcal{W} = \{w_1, w_2, ..., w_K\}$ and $\mathcal{R} = \{r_1, r_2, ..., r_K\}$ denote the sets of group winners and runners-up, respectively. Let $\mathcal{V}$ be the set of all possible perfect matchings between $\mathcal{W}$ and $\mathcal{R}$. We examine a random mechanism $\psi : 2^\mathcal{V} \to \Delta \mathcal{V}$ that takes as an input any subset $\Gamma$ of $\mathcal{V}$ (a set of admissible matchings) and as an output induces a

\(^{14}\)Online audiences have increased for the draw ceremony over time. In the 2015 season the R16 draw was watched by 262,000 viewers, in the 2016 season by 415,000, and in the 2017 season by 618,000.

\(^{15}\)See Table 7 in the Appendix for the constraints in seasons 2004–18.
probability distribution over the elements of $\Gamma$. The mechanism works through the following procedure:

**Algorithm** (Constrained $R$-to-$W$ dynamic draw). *Given an input set of admissible matchings $\Gamma \subseteq \mathcal{V}$, the algorithm selects a matching $\psi(\Gamma)$ in $K = |W|$ steps.*

**Initialization:** Set $R_0 = R$, and $\Gamma_0 = \Gamma$.

**Step-$k$:** (for $k = 1$ to $K$)

(i) Randomly choose $v^R_k \in R$ through a fair draw over $R_{k-1}$;

(ii) Randomly choose $v^W_k \in W$ through a fair draw over the set of admissible partners for $v^R_k$, $\mathcal{W}_k := \{w \in W | \exists V \in \Gamma_{k-1} \text{ s.t. } v^R_k w \in V\}$ ;

(iii) Construct the sets of currently unmatched runners-up $R_k = R_{k-1} \setminus \{v^R_k\}$ and valid assignments given current draw $\Gamma_k = \{V \in \Gamma_{k-1} | v^R_k v^W_k \in V\}$.

**Finalization:** After $K$ steps the algorithm returns a vector of $K$ match pairs

$$\mathbf{v} = (v^K_1 v^W_1, \ldots, v^K_K v^W_K).$$

The unordered matching $\{v^K_1 v^W_1, \ldots, v^K_K v^W_K\}$ is set as the realization of $\psi(\Gamma)$. 

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**Figure 1.** Possible matchings against the number of same-nation exclusions (2004–18)
This dynamic procedure initially produces an ordered sequence of \( K \) matches, \( v \). In order to characterize the probability of the matching \( V \) we define: (i) \( \mathcal{P}(V) \), the set of all possible order-permutations for the matches in \( V \); and (ii) \( \mathcal{W}_k(v) \), the set of admissible match partners for runner-up \( v_k^R \) selected at Step-\( k(i) \) in the permutation \( v \). \(^{16}\)

**Proposition 1.** Under the constrained \( R \)-to-\( W \) draw the probability of any matching \( V \in \Gamma \) is given by

\[
\Pr \{ \psi(\Gamma) = V \} = \frac{1}{K!} \sum_{v \in \mathcal{P}(V)} \prod_{k=1}^{K} \frac{1}{\mathcal{W}_k(v)}.
\]

**Proof.** For any matching \( V \in \Gamma \), each of the \( K! \) possible permutations of \( V \) has a strictly positive probability, so \( \Pr \{ V \} = \sum_{v \in \mathcal{P}(V)} \Pr \{ v \} \), which can be rewritten using the chain rule as

\[
\Pr \{ V \} = \sum_{v \in \mathcal{P}(V)} \prod_{k=1}^{K} \left( \Pr\{v_k^R | v_{k-1}\} \cdot \Pr\{v_k^W | v_k^R, v_{k-1}\} \right),
\]

where \( v_{k-1} \) denotes the matches selected in steps 1 through \( k - 1 \). Since the randomization is fair at each step, \( \Pr\{v_k^R | v_{k-1}\} \) simplifies to \( \frac{1}{K-k+1} \) and \( \Pr\{v_k^W | v_k^R, v_{k-1}\} = \frac{1}{\mathcal{W}_k(v)} \). \( \square \)

Proposition 1 shows that the mechanism’s probability distribution over \( \Gamma \) requires \( K! \times |\Gamma| \) calculations. Even though the cardinality of \( \Gamma \) can be substantially lower than \( K! \), the exact computation of \( \Pr \{ V \} \) involves between \( K! \) and \( (K!)^2 \) steps, and can be taxing even for our application with \( K = 8 \).

Though the above assignment rule is combinatorically involved, the UEFA draw procedure has three useful features. First, all draws are conducted using an urn, and thus the randomization is fair. Second, the number of realizations from each draw is less than eight, implying that the draw is easy to comprehend and timely. Finally, all nontrivial elements of the draw conducted opaquely by the computer (the combinatoric calculation of the set of valid partners) can be checked ex post, and thus the draw is fully verifiable. Consequently, as long as the urn draws are conducted fairly and publicly, \(^{17}\) it is not possible for the designer to cheat within the procedure, as any deviation from the prescribed algorithm along the path of play is observable and checkable by interested parties/observers. This inoculates the one opaque part of the process from corruption by the principals.

\(^{16}\)That is: \( \mathcal{W}_k(v) := \left| \{ w \in \mathcal{W} \mid \exists V \in \Gamma \text{ s.t. } v_k^R w \in V \text{ and } \bigwedge_{j=1}^{k-1} (v_j^R v_j^W \in V) \} \right| \).

\(^{17}\)Unlike many state lotteries which use mechanical randomization devices to draw urn outcomes, the UEFA draw is conducted by human third-parties (typically famous footballers). Pointing to the football fans’ distrust in the randomizer, this has led to plausible allegations of UEFA rigging draws with hot/cold balls (here made by former FIFA president Sepp Blatter in an interview with Argentine newspaper *La Nacion*, June 13th, 2016).
Given the characterization in Proposition 1, one question is the extent to which the above calculation can be simplified. We define two randomizations as being equivalent if they produce the same distribution over matchings $\mathcal{V}$, and distinct if they differ.

**Proposition 2.** The constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw is distinct from:

(i) The constrained uniform draw over $\Gamma$.

(ii) The constrained dynamic procedure that fairly draws admissible pairs.

(iii) The constrained $\mathcal{W}$-to-$\mathcal{R}$ dynamic draw.

Proof. See Appendix for counter-examples. □

The first two parts of the Proposition 2 are essentially negative results, indicating that our environment is not equivalent to simpler fair draws over matchings or individual matches, where the third part shows the procedure is asymmetric. While the main takeaway from Proposition 2 is negative, it does demonstrate three potentially constructive design channels. For instance, the ability to vary outcomes by choosing to draw students first and schools second or vice versa in a school choice setting might be leveraged to obtain more-desirable outcomes. See for instance the reversal of the proposing sides in the National Resident Matching Program (NRMP) algorithm detailed in Roth and Peranson (1997).\(^{18}\)

### 4.2. Relaxing the UEFA Constrained Assignment Problem.

Above we provide the general structure of the dynamic mechanism employed by UEFA in the R16 match. Within this family of procedures, the specific draw defines the admissible matchings $\Gamma$ via a set of match exclusions $H \subset \mathcal{R} \times \mathcal{W}$, where the overall exclusion set $H = H_A \cup H_G$ is the union of the association-level exclusions $H_A$ and group-level exclusions $H_G$. The precise set $H$ varies across seasons depending on the composition of teams reaching the R16 stage and the seeding prior to the draw. The admissible matching set for the UEFA implementation of the constrained dynamic draw is defined by

$$\Gamma_H := \{V \in \mathcal{V} | V \cap H = \emptyset\},$$

where the draw induces the random matching $\psi(\Gamma_H)$.

However, while distinct, we later show that in our particular setting the three draw procedures lead to only marginally different outcomes, where the fair draw over $\Gamma$ has a tractability advantage for approximating the assignment probabilities. The standings of the UCL groups are fixed two days before the draw is carried out. While analytic calculation of the exact matching probabilities for the draw would take approximately two months on a powerful desktop, probabilities for the fair draw over feasible assignments can be calculated in fractions of second.
While our paper later discusses and quantifies how the specific constraints in $H$ affect the expected draw outcomes, we also examine the extent to which fairer random assignments might exist. To aid us in this endeavor we employ the core result in Budish et al. (2013) that guarantees the existence of an equivalent randomization over feasible assignments for every feasible expected assignment. This allows us to relax the constrained assignment problem over discrete final matches to the one of finding expected assignments allowing for fractional (and continuous) assignments of objects.

First, notice that any assignment $V$ can be rewritten as a matrix $X(V) \in \{0, 1\}^{K \times K}$ with a generic entry $x_{ij}(V) = 1 \{r_i w_j \in V\}$ indicating whether or not runner-up $r_i$ is matched to winner $w_j$. Since $V$ represents a perfect matching between $R$ and $W$, $X(V)$ is a rook-matrix where each row and column have exactly one non-zero entry. Second, for any random draw over the feasible assignment set $\Gamma_H$, we define the expected assignment matrix as $A = \mathbb{E}X(V) = \sum_{V \in \Gamma_H} \Pr \{V\} \cdot X(V)$ with a generic entry $a_{ij}$ representing the likelihood of $(r_i w_j)$ match-up. Finally, we define an expected assignment matrix $A$ as satisfying the UEFA constraints if: $\forall ij \in H : a_{ij} = 0$; $\forall i \in R, j \in W : 0 \leq a_{ij} \leq 1$, $\sum_{k=1}^{K} a_{kj} = \sum_{k=1}^{K} a_{ik} = 1$. That is, for an expected assignment matrix $A$ to satisfy the constraint set, each of its entries must be a non-negative real number representing a probability, all rows and columns must sum to exactly one, and the probability of the excluded matches must be equal to zero. While satisfying the UEFA constraints is clearly a necessary condition for any expected assignment resulting from a randomization over feasible assignments, the below shows its sufficiency:

**Proposition 3** (Implementability). Any expected assignment matrix $A$ satisfying the UEFA constraints is implementable through some randomization over the constrained assignment set $\Gamma_H$.

**Proof.** The UEFA constraints can be grouped into two distinct sets: (i) the singleton exclusions $H$ and the $K$ row constraints; and (ii) the $K$ column constraints. As such the UEFA constraints satisfy the biheirarchy condition in Theorem 1 from Budish et al. (2013). □

This result indicates that instead of analyzing distributions over $\Gamma_H$ (with $O(K!)$ degrees of freedom), we can focus on investigating the properties of expected assignments (with $O(K^2)$ degrees of freedom). Accordingly, for our specific applications with $K = 8$, Proposition 3 allows us to optimize with just 30–40 degrees of freedom rather than with 2,000–10,000.

A trivial corollary to the above result is that:

$^{19}$The quotas for each element $a_{ij}$ are therefore a min and a max of zero for the excluded singletons; a min of zero and a max of one for the non-excluded singletons; a min and a max of one for the row sum; and a min and a max of one for the column sum.
Corollary 1. Any expected assignment matrix satisfying the group and association constraints is implementable by randomizing over a finite collection of $J$ constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draws $\{\psi(\Gamma_j)\}_{j=1}^J$, where each $\Gamma_j \subseteq \Gamma_H$.

Proof. Set $J = |\Gamma_H| < K!$ and for each entry $V_j \in \Gamma_H$ set $\Gamma_j = \{V_j\}$. By Proposition 3, there exists a probability $p_j$ of selecting each admissible matching $V_j$ that leads to any implementable expected assignment matrix. The result follows from setting $\Pr\{\Gamma_j\} = p_j$. □

While Proposition 3 facilitates the examination of whether better randomizations might exist, Corollary 1 provides us with a direct tool for implementation by randomizing over the input set of admissible assignments.

5. Quantifying the Constraint Effects

In this section, we first provide examples of how the current UEFA mechanism affects expected assignments in the UCL R16, and how the spillovers between the same-nation exclusions impact equally seeded teams. Next, we examine the extent to which fairer randomizations are possible. After showing that substantially better mechanisms do not exist, in our final analysis we turn to the extent to which gains can be made by slightly relaxing the underlying constraint set.

5.1. Distortions in the Current UEFA Mechanism. We start our empirical analysis with an illustrative example from the UCL R16 draw in the 2018 season. The corresponding expected assignment matrix under the UEFA dynamic draw is given in Table 1. Each row represents a group winner, and each column a runner-up. In row $i$ and column $j$, we provide the probability (calculated by Monte Carlo simulation) that the realized R16 matching contains the $(ij)$-pair.\(^{20}\)

The constraints in the 2018 draw are as follows: First, along the diagonal, the probabilities of each match are zero, reflecting the eight group constraints. Second, seven same-nation matches are excluded reflecting the association constraint. Finally, all rows and columns sum to exactly one, as each represents the marginal match distribution for the respective team, through the bipartite constraint. \(^{21}\)

\(^{20}\)Given our simulation size of $N = 10^6$, 95 percent confidence intervals for each coefficient are contained in a ball of radius 0.001 (see Proposition 4 in the Appendix).

\(^{21}\)Note that the constraints are not mutually exclusive and consequently, even though the expected assignment is an $8 \times 8$ matrix, it has 34 degrees of freedom. The expected assignment matrices for the R16 draw in the other 14 seasons can be found in the paper’s Online Appendix.
Table 1. for the 2018 R16 draw

<table>
<thead>
<tr>
<th></th>
<th>Basel</th>
<th>Bayern Munchen</th>
<th>Chelsea</th>
<th>Juventus</th>
<th>Sevilla</th>
<th>Shakhtar Donetsk</th>
<th>Porto</th>
<th>Real Madrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manchester United</td>
<td>0.0</td>
<td>0.148</td>
<td>0.0</td>
<td>0.183</td>
<td>0.183</td>
<td>0.155</td>
<td>0.148</td>
<td>0.182</td>
</tr>
<tr>
<td>Paris Saint-Germain</td>
<td>0.109</td>
<td>0.294</td>
<td>0.128</td>
<td>0.128</td>
<td>0.108</td>
<td>0.105</td>
<td>0.105</td>
<td>0.128</td>
</tr>
<tr>
<td>Roma</td>
<td>0.159</td>
<td>0.151</td>
<td>0.0</td>
<td>0.189</td>
<td>0.16</td>
<td>0.152</td>
<td>0.189</td>
<td>0.0</td>
</tr>
<tr>
<td>Barcelona</td>
<td>0.149</td>
<td>0.144</td>
<td>0.413</td>
<td>0.0</td>
<td>0.15</td>
<td>0.144</td>
<td>0.128</td>
<td>0.105</td>
</tr>
<tr>
<td>Liverpool</td>
<td>0.159</td>
<td>0.151</td>
<td>0.0</td>
<td>0.189</td>
<td>0.16</td>
<td>0.152</td>
<td>0.189</td>
<td>0.183</td>
</tr>
<tr>
<td>Manchester City</td>
<td>0.156</td>
<td>0.148</td>
<td>0.0</td>
<td>0.183</td>
<td>0.184</td>
<td>0.148</td>
<td>0.129</td>
<td>0.0</td>
</tr>
<tr>
<td>Besiktas</td>
<td>0.109</td>
<td>0.105</td>
<td>0.293</td>
<td>0.128</td>
<td>0.128</td>
<td>0.108</td>
<td>0.0</td>
<td>0.129</td>
</tr>
<tr>
<td>Tottenham Hotspur</td>
<td>0.16</td>
<td>0.152</td>
<td>0.0</td>
<td>0.189</td>
<td>0.189</td>
<td>0.159</td>
<td>0.151</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*Note: Probabilities derived from a simulation \((N = 10^6)\) of the UEFA draw procedure.*

Despite having a fair draw from an urn at each point in time, the likelihoods of two teams playing each other are not uniform. For illustration, consider Paris Saint-Germain (PSG) in the second row of Table 1. As PSG is the only French team in the R16 in the 2018 season it has no same-nation exclusions and thus, seven potential match partners. However, the likelihoods of the different match-ups vary substantially. For example, the probability of PSG playing Chelsea is almost three times larger than that of playing Basel, Shaktar, or Porto.

In what follows, we quantify the effects of distortions induced by the constraints in the R16 matching. First, we introduce a model for counterfactual game outcomes, which we estimate using historical (and out-of-sample) data for each UCL season from 2004 to 2018. After outlining our data and discussing the estimation procedure, we quantify the total effects of the constraints using the estimated model parameters and the tournament’s prize structure. By simulating the R16 draws and the subsequent games within the tournament, we show that tournament matching constraints have major effects on teams’ expected outcomes. Finally, we set out an objective function designed to isolate the indirect spillovers from enforcing the constraints over the space of expected assignments. We demonstrate that even though the spillovers are substantial (especially in seasons with many same-nation exclusions) the UEFA draw procedure comes very close to a constrained-best.

5.2. Data and Estimation of Game-Outcome Model. In order to account for the variation in teams’ ability while examining potential effects driven by the tournament’s constraints, we estimate an off-the-shelf structural model from the sports economics literature: the bivariate Poisson (Maher, 1982; Dixon and Coles, 1997).

**Model.** Let \(S_i\) and \(S_j\) be the number of goals scored by the home team \(i\) and guest team \(j\) in a given game. In a bivariate Poisson model, \((S_i, S_j) \sim (\lambda_1, \lambda_2, \lambda_3)\), the joint probability
function is of the form
\[
P_{(S_i, S_j)}(s_i, s_j) = \exp \left\{ - (\lambda_1 + \lambda_2 + \lambda_3) \right\} \frac{\lambda_1^{s_i} \lambda_2^{s_j} \min(s_i, s_j)}{s_i! s_j!} \sum_{k=0} \binom{s_i}{k} \binom{s_j}{k} k! \left( \frac{\lambda_3}{\lambda_1 \lambda_2} \right)^k,
\]
where \(E(S_i) = \lambda_1 + \lambda_3, E(S_j) = \lambda_2 + \lambda_3\) and \(\text{Cov}(S_i, S_j) = \lambda_3\).

In our estimation we follow Karlis and Ntzoufras (2003) and assume that \(\ln \lambda_1 = \mu + \eta + \alpha_i - \delta_j\), \(\ln \lambda_2 = \mu + \alpha_i - \delta_j\), and \(\lambda_3 = \rho\), where \(\mu\) denotes a season-specific constant term, \(\eta\) a home-effect parameter, and \(\alpha_k\) and \(\delta_k\) are the attack and defense parameters measuring the idiosyncratic offensive and defensive performance of team \(k\).

We estimate the above bivariate Poisson model separately for each season \(t\) between 2004 and 2018 via constrained maximum likelihood. For scale identification we impose two sum-to-zero constraints, and assume that \(\sum_k \alpha_k = \sum_k \delta_k = 0\). In the estimation, we use scoreline data from the group stage in season \(t\) together with game-level data from the group and knockout stages (except for the final game which is played on a neutral soil) in seasons \(t-1\) and \(t-2\). This results in a total of 408 game-level observations used in the 2004 estimation, 376 observations for the 2005 season, and 348 observations for each season between 2006 and 2018.\(^{22}\) In Table 2, we provide a brief summary of the statistics for the number of goals scored by the home and away teams across the 17 most recent UCL seasons. On average, home teams score more goals than away teams, where the average number of goals for both teams increases over time. Moreover, we observe a substantial increase in the uncertainty of the game’s outcome between the 2002 and 2018 seasons.

In Figure 2, we graph the estimated defense parameters on the horizontal axis and the corresponding estimated attack parameters on the vertical axis (for all R16 teams across all seasons between 2004 and 2018). The strongest teams have large positive values for both the attack and defense parameters; see for example FC Barcelona (2011) or Chelsea FC (2010) in the first quadrant. Alternatively, the low-performing teams have either a negative value of the attack parameter (low offense) or a negative value of the defense parameter (low defense); see FC Lokomotiv Moskva (2004) in the second quadrant or AC Sparta Praha (2004) in the fourth quadrant. Teams of low-to-medium-strength with small but positive values of both attack and defense parameters are centered at zero in the first quadrant.\(^{23}\)

In Table 3, we present the summary statistics for the estimated attack and defense parameters by the stage of the competition. Teams advancing to the semifinals have both\(^{22}\) The differences in the number of observations result from the change to the tournament design in the 2004 season, where the second group stage directly preceding the quarterfinals was replaced by the R16. Consequently, the number of games played was reduced from 48 to 16.\(^{23}\) See Table 8 in the Appendix for the estimates of the constant term, the home-effect parameter, and the correlation coefficient between the number of goals scored by opposing teams in seasons 2004–18.
TABLE 2. Summary statistics for goals scored from the group stage onward (2002–18)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>156</td>
<td>1.69</td>
<td>0.95</td>
<td>1.18</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>2003</td>
<td>156</td>
<td>1.58</td>
<td>1.19</td>
<td>1.36</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>2004</td>
<td>124</td>
<td>1.52</td>
<td>0.94</td>
<td>1.37</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>2005</td>
<td>124</td>
<td>1.69</td>
<td>0.97</td>
<td>1.46</td>
<td>1.07</td>
<td>1.07</td>
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<tr>
<td>2006</td>
<td>124</td>
<td>1.39</td>
<td>0.86</td>
<td>1.31</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>2007</td>
<td>124</td>
<td>1.47</td>
<td>1.00</td>
<td>1.29</td>
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<td>1.05</td>
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<tr>
<td>2008</td>
<td>124</td>
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<td>1.07</td>
<td>1.42</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>2009</td>
<td>124</td>
<td>1.45</td>
<td>1.19</td>
<td>1.34</td>
<td>1.28</td>
<td>1.28</td>
</tr>
<tr>
<td>2010</td>
<td>124</td>
<td>1.42</td>
<td>1.15</td>
<td>1.23</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td>2011</td>
<td>124</td>
<td>1.64</td>
<td>1.19</td>
<td>1.44</td>
<td>1.27</td>
<td>1.27</td>
</tr>
<tr>
<td>2012</td>
<td>124</td>
<td>1.68</td>
<td>1.09</td>
<td>1.54</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td>2013</td>
<td>124</td>
<td>1.63</td>
<td>1.31</td>
<td>1.30</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>2014</td>
<td>124</td>
<td>1.62</td>
<td>1.26</td>
<td>1.35</td>
<td>1.33</td>
<td>1.33</td>
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<tr>
<td>2015</td>
<td>124</td>
<td>1.70</td>
<td>1.19</td>
<td>1.60</td>
<td>1.33</td>
<td>1.33</td>
</tr>
<tr>
<td>2016</td>
<td>124</td>
<td>1.68</td>
<td>1.12</td>
<td>1.42</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td>2017</td>
<td>124</td>
<td>1.84</td>
<td>1.19</td>
<td>1.68</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>2018</td>
<td>124</td>
<td>1.77</td>
<td>1.43</td>
<td>1.53</td>
<td>1.40</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Note: From the left to the right: UCL season, total number of games played, average number of goals scored by home and guest teams, and standard deviation of the number of goals scored by home and guest teams.

Figure 2. Estimated attack and defense parameters for the UCL teams (2004–18)
Table 3. Summary statistics for estimated parameters

<table>
<thead>
<tr>
<th>Stage</th>
<th>Attack parameter</th>
<th>Defense parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Med.</td>
</tr>
<tr>
<td>R16</td>
<td>0.68</td>
<td>0.60</td>
</tr>
<tr>
<td>QF</td>
<td>0.73</td>
<td>0.67</td>
</tr>
<tr>
<td>SF</td>
<td>0.78</td>
<td>0.74</td>
</tr>
</tbody>
</table>

better offensive and defensive performance relative to those competing in the quarterfinals. Likewise, teams that move on to the quarterfinal matches are stronger both offensively and defensively relative to R16 teams. Moreover, the two-sample Kolmogorov-Smirnov test suggests that the empirical distributions of attack and defense parameters in the R16 and the semifinals are statistically different (at the 10 percent significance level).

5.3. Constraint effects in the UCL draw. In what follows, we quantify the total and spillover effects from the constraints on teams’ outcomes (such as expected prize money and the probability of advancing to the later stages of the competition).

**Result 1.** The association constraint in the R16 generates substantial effects: (i) altering expected tournament prizes by millions of euro; (ii) significantly affecting the chances of reaching later stages of the tournament; and (iii) creating spillovers to the matching chances of otherwise equally treated outcomes.

**Evidence:** We carry out two simulations of the UEFA draw mechanism that differ in the underlying set of restrictions using the estimated parameters of the goal-outcome model. In the first simulation we impose all three constraints (the bipartite, group, and association constraints), whereas in the second simulation, we drop the association constraint. In each simulation, we first randomly draw $J = 1,000$ R16 matchings $\{V^t_j\}_{j=1}^J$ under the relevant admissible sets in each tournament year $t$ (fixing the realized R16 teams in season $t$). Next, for each draw $V^t_j$ we simulate the remaining games in the tournament $S = 1,000$ times (the R16 home/away games, quarter- and semifinal home/away games, and the final game on neutral soil). Given the simulations, we calculate team $i$’s expected earnings in season $t$ conditional on the draw $V^t_j$ and denote them as $\hat{E}π^A_{itlj}$ and $\hat{E}π^B_{itlj}$ under the current UEFA mechanism and the counterfactual mechanism that drops the same-nation exclusions, respectively.

In Figure 4(A) on the horizontal axis, we graph the simulated expected prize money under the current UEFA draw, $\hat{E}π^A_{itlj} = \frac{1}{J} \sum_{j=1}^J \hat{E}π^A_{itlj}$ for all R16 teams $i$ in all seasons $t$ by averaging across the $J$ draws; on the vertical axis, we graph the difference in simulated prize money
Figure 3. Simulated expected prize money versus realized prize (2018)

between the 10th and 90th percentiles of the empirical distribution of \( \{ \hat{E}_{\pi_{t|j}} \} \) (for all R16 teams \( i \) across all seasons \( t \)). The figure thereby indicates the strong link between the variability in R16 draw realizations and team’s expected prize money. For clubs most likely to be eliminated in the R16 (and so earning less than 20 million euro in expectation) the prize money difference between the most and least favored draws does not exceed a quarter million euro. Alternatively, for teams most likely to compete in the championship game (and so earning more than 30 million euro in expectation), the prize money difference is three times larger. We conclude that the realization of the R16 draw is of higher importance to the best-performing teams in the competition.

Looking across all 15 seasons, the expected prize money is strongly correlated with the realized prize \( (\rho = 0.639) \). From a linear regression on 240 team-year observations (16 teams across 15 seasons), we find that the simulated prizes are highly predictive of the realized prizes \( (p < 0.000) \); where Figure 3 provides illustration of a relationship between the simulated and realized tournament prize money in the 2018 season.\(^{24}\)

While Figure 4(A) illustrates the variability in prize money outcomes induced by draw realizations, Figure 4(B) shows a complementary view on the simulated probability of reaching the semifinals of the competition. A favorable R16 draw increases the chance of an upper-quartile-team advancing to the semifinals by about five percentage points. However, even a

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\(^{24}\)Our bivariate Poisson model explains 41 percent of the total variation in prize money.
best-case draw does not increase the chance for a lower-quartile-team by more than a percentage point. Across our analyses, similar results follow for alternative metrics. We choose, however, to focus on prize money as an objective measure of how the tournament organizers weigh reaching the later stages of the competition.

In order to measure the distortions caused by the association constraint, we calculate the expected prize difference as $\Delta \pi_i^t := \hat{E}\pi_{it}^A - \hat{E}\pi_{it}^B$ (where $\hat{E}\pi_{it}^B = \frac{1}{J} \sum_{j=1}^{J} \hat{E}\pi_{it|j}$ is the expected payoff under the dynamic draw that allows same-nation matches). Teams with a positive value of $\Delta \pi_i^t$ are those benefiting from the association constraint, whereas those with a negative value are being disadvantaged. In what follows, we refer to $\Delta \pi_i^t$ as the total constraint effect. Across all 15 UCL seasons, the total constraint effect has a standard deviation of 0.3 million euro (it is mean-zero by construction) and a range of 1.8 million euro (a loss of -0.8 million euro for Barcelona in the 2007 season and a gain of 1.0 million euro for Real Madrid in the 2017 season).

To validate the above as being driven by the association constraint, we explain the variation in teams’ outcomes orthogonal to their ability by constructing seasonal ability indexes. First, for each team $i$ in season $t$, we calculate the probability of winning against each of the other 15 teams participating in that year’s R16 using the estimated bivariate Poisson model parameters and assuming play on a neutral ground. Second, we average over the fifteen win probabilities,

$$\omega_i^t = \frac{1}{15} \sum_{j \neq i} Pr(S_i > S_j; \eta = 0, \hat{\mu}, \hat{\rho}, \hat{\alpha}_i, \hat{\delta}_i, \hat{\alpha}_j, \hat{\delta}_j).$$

to construct team $i$’s ability index in season $t$. Finally we linearly re-scale the figures for each season to run from zero to one: $\bar{\omega}_i^t = (\omega_i^t - \bar{\omega}_i^t) / (\bar{\omega}_i^t - \bar{\omega}_i^t)$. We refer to constructed $\bar{\omega}_i^t$ as team $i$ ability index in season $t$.25

In Figure 5, we illustrate the relationship between the total constraint effect $\Delta \pi_i^t$ and the residuals from a regression of realized tournament prizes on the ability index $\bar{\omega}_i^t$.26 Even though the constructed constraint effect explains only 4.7 percent of the total variation in realized prizes not accounted for by the team’s ability, it is statistically significant at any conventional significance level ($p < 0.001$). A probit model examining whether a team advances to the semifinals of the competition, with the ability index and the total constraint effect as explanatory variables, suggests that both are highly significant. While the ability effect increases the likelihood of a semifinal appearance by an average of 67.3 percent ($p < 0.000$),

25For example, in the 2018 season, the ability index runs from Shaktar Donetsk at 0, FC Basel at 0.134 and Besiktas at 0.233, up to Real Madrid at 0.919, Liverpool at 0.999, and Barcelona at 1.
26Our ability index coefficient indicates that 17.8 million euro of the realized prizes can be explained by team’s ability.
Note: Both panels show simulated expected outcomes against simulated differences between the 10th and 90th percentiles of the corresponding empirical distributions. In panel (A) the variable of interest is the prize money, whereas in panel (B) the variable of interest is the probability of reaching the tournament’s semifinals.

Figure 4. Unconditional expected outcomes against variability in draw outcome.

(A) Difference in Simulated Prize Money (€ millions)

(B) Difference in Simulated Probability of Reaching SFs
Figure 5. Constraint effect versus realized tournament prize residual

Note: The association constraint effect $\Delta \pi_t^i$ is the simulated effect on prizes from enforcing the association constraints. The prize residual is the difference between the realized tournament prize and the estimated one. Dashed line shows fitted relationship, with band showing 95 percent confidence on relationship.

Above, we quantify the total effects of the association constraint, consisting of direct and indirect spillovers affecting both constrained and unconstrained teams. For illustration, consider match probabilities of Real Madrid and Barcelona in the 2018 season (see Table 1, column 8 and row 4, respectively). While the Read-Madrid and Barcelona exclusion directly benefits the two Spanish teams (which are the two of the three strongest according to our ability index in the 2018 season) Barcelona additionally profits from substantial spillovers generated by six other same-nation exclusions. In particular, consider Barcelona and the two unconstrained group winners, Basel and Shaktar Donetsk (each with seven potential match partners, rows 2 and 7, respectively). Even though none of the three group winners are constrained from matching with the two lowest-performing teams (Basel and Shaktar Donetsk, columns 1 and 6) Barcelona is 1.35 times more likely to match to either of them than are the unconstrained group winners, resulting in inequality across equals.

27The constraint effects on reaching later stages of the tournament are diminishing. Whereas a one million euro gain from the association constraint increases the chances of reaching the quarterfinals by 31.2 percent ($p = 0.001$), the chances of reaching the final are raised by only 9 percent ($p = 0.086$).
In order to quantify the indirect effects of the association constraint we design a spillover measure (built on the domain of expected assignment matrices to make later use of Proposition 3) Specifically, our objective is motivated by the idea of equal treatment of equals (ETE). We regard two teams $i$ and $j$ as equal if they are both unconstrained from a match with team $k$, and as more equally treated the closer are the likelihoods of the $ik$ and $jk$ matches. In this way, the objective precisely gets to the indirect effects of the constraints. For any expected assignment $A$ our ETE objective measures the mean absolute difference between equally treated team’s match likelihoods:

$$Q(A) = \frac{1}{|\Upsilon|} \sum_{(ik,jk) \in \Upsilon} |a_{ik} - a_{jk}|,$$

where $\Upsilon = \{(ik,jk) | i, j \in W, k \in R, ik, jk \notin H\} \cup \{(ki,kj) | k \in W, i, j \in R, ki, kj \notin H\}$.

**Figure 6.** Values of the equal-treatment-of-equals objective function against the number of same-nation exclusions (2004–18)

In Figure 6, we graph the ETE objective $Q(A)$ for the current UEFA draw procedure against the number of same-nation exclusions by season. Since the relationship in question is strongly positive, we conclude that the association constraint substantially distorts match chances of equal teams.\(^{28}\) Specifically, the estimated slope coefficient from the regression of $Q(A)$ on the number of same-nation exclusions suggests an average wedge in match-likelihoods for equally treated teams of 0.05 for every ten exclusions. This represents a relative swing of up to a third for unconstrained teams. Although this result points to quantitatively large

\(^{28}\)When there are no same-nation exclusions our measure is zero by construction.
spillovers even after accepting the constraints’ direct effect, we now show that there is not much scope to ameliorate the spillovers through better randomization.

5.4. Near-Optimality of the Current Procedure. A natural question raised by Result 1 is whether there exists a randomization procedure that generates less distortions than the current procedure. In this section we provide a largely negative answer:

**Result 2.** While the UEFA mechanism is not optimal given the constraints, it comes very close to the optimal mechanism when considering equal treatment of equals.

**Evidence:** To begin with, we turn back to Proposition 3 in Section 4.2, where we show that a feasible assignment-producing mechanism exists for every feasible expected assignment satisfying the constraints. This substantially simplifies the problem of finding better mechanisms as it enables us to search in the space of expected assignments. Our main argument for Result 2 is that the optimal expected assignment \( \mathbf{A}^*_t \) is not substantially better than the expected assignment under the current draw \( \hat{\mathbf{A}}^*_t \), for any season from 2004 to 2018. We numerically assess the expected assignment that solves the following optimization problem:

\[
\mathbf{A}^*_t := \arg\min_{\mathbf{A}} Q(\mathbf{A}),
\]

subject to the constraints (1) \( \forall ij \in H_t : a_{ij} = 0 \); (2) \( \forall ij : 0 \leq a_{ij} \leq 1 \); (3) \( \forall i : \sum_k a_{ik} = \sum_k a_{ki} = 1 \), where \( H_t \) denotes the set of match exclusions in season \( t \) and \( Q \) is the ETE objective function defined in Section 5.1.

In Figure 7, we graph values of the best-case ETE objective \( Q(\mathbf{A}^*_t) \), against the values of the objective for the actual UEFA mechanism across all seasons. While some improvement is possible across the tournament years, the gains are marginal (on average a less than 10 percent reduction in the spillover measure \( Q \)).

Against the small potential benefits, there are large potential complexity costs associated with modifying the existing mechanism. Namely, mechanisms that produce the optimal expected assignment are complex in comparison to the current procedure. While Corollary 1 provides a channel through which the optimal mechanism might be implemented—a pre-stage draw where the organizers randomize the feasible matchings—even this would be cumbersome and might engender suspicion from observers. Put against this cost, the ability to reduce the average match distortion from 5 percent using the current UEFA mechanism to 4.5 percent using the optimal mechanism does not seem alluring.

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29 The size of the reduction is given by the estimated slope coefficient from the regression of the optimal ETE objective against the objective for the actual UEFA mechanism for all years \( t = 2004, \ldots, 2018 \).

30 Note that it is possible that there exist simple modifications of the current matching mechanism that would shift the expected assignments towards the optimum. In the Appendix, we show that the three
Figure 7. Values of the equal-treatment-of-equals objective function for the UEFA mechanism against the optimal mechanism subject to constraints.

Note that the inability to improve upon the expected assignments generated under the UEFA mechanism is not driven by a limited scope in moving the expected assignments given the constraints. For illustration, consider the 2018 season, and assume that the probability of any unconstrained pairing is equal to either zero or one. This deterministic (and valid) assignment would result in the value of the ETE objective of 0.322. This worst-case outcome is both attainable in all cases and substantially greater than the value of the ETE objective under the current UEFA mechanism.

5.5. Weakening the Constraints. One response to the Result 2 is to accept the current mechanism and the distortions it generates, because we cannot practically do better. In this section, however, we take a different approach, and examine the extent to which gains can be made by weakening the association constraint. Specifically, we investigate the extent to which the problem can be partially relaxed while still protecting the tournament from excessive same-nation match-ups. There are several ways in which this could be accomplished. In what follows, we focus solely on a procedure that weakens the association constraint but at the same time continues to use a draw procedure similar to the current one. Specifically, we study the family of pure assignments that allow at most one same-nation match in the R16. We continue to use the constrained $\mathcal{R}$-to-$\mathcal{W}$ dynamic mechanism detailed in Section 4.1, but distinct mechanisms set out in Proposition 2 are all nearly identical in their expected assignments to the actual mechanism (see Figures 11-13 in the Appendix).
we expand the admissible set to
\[ \Gamma_H := \{ V \in \mathcal{V} \mid |V \cap H| \leq 1 \} . \]

As such, the relaxation retains the desirable features of the mechanism; the randomization is completely transparent (fair draws from a small-sample urn) and the more-opaque combinatoric check continues to be fully verifiable at all points during the draw.

In what follows we find that:

**Result 3.** Weakening the association constraint to allow for at most one same-association match in the R16 substantially reduces the distortions, while protecting associations from excessive same-nation match-ups. Moreover, as a secondary effect, weakening the association constraint reduces the number of same-nation games in the later stages of the tournament.

**Evidence:** We start by constructing analog results to those presented in Section 5.1. In Figure 8, we graph values of the ETE objective of the UEFA draw procedure with at-most-one same-association constraint against that of the UEFA procedure with the original constraint set. Allowing for at most one same-nation match in the R16 decreases the distortions by more...
than 70 percent. This is a sizable reduction, especially when compared to the 10 percent reduction obtained under the optimal mechanism derived in Section 5.4.

Next, we calculate the expected prize difference as $\Delta \bar{\pi}_i^t = \tilde{\pi}_1^t - \tilde{\pi}_0^t$, where $\tilde{\pi}_1^t$ and $\tilde{\pi}_0^t$ are team $i$'s simulated average earnings in season $t$ under the at-most-one same-association match mechanism and the counterfactual mechanism where the association constraint is entirely removed, respectively. In Figure 9, we illustrate the relationship between $\Delta \bar{\pi}_i^t$, which we refer to as the total counterfactual constraint effect, and the total constraint effect $\Delta \pi_i^t$ defined in Section 5.1. Overall, we conclude that allowing for a single same-association match in the R16 reduces the prize distortions by 71 percent.

Above we focus on the benefits from reducing distortions caused by the association constraint. However, there are presumably nontrivial costs associated with allowing for same-nation matches. Relaxing the association constraint as we have done leads to a single same-nation match-up in the R16 in approximately six out of every ten tournaments. In seasons with at least six same-nation exclusions, this ratio increases to seven-in-ten. Moreover, since the

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31 The reduction is given by the estimated slope coefficient from a regression of the ETE objective under the at-most-one same-association draw against the objective for the actual UEFA draw, across seasons $t = 2004, ..., 2018$.

32 Estimated slope coefficient of 0.29 from the regression of the direct counterfactual association effect on the current association effect, for all teams in all seasons.
association constraint is imposed intentionally, UEFA likely has a clear underlying preference for the tournament to be primarily an international competition.\textsuperscript{33}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure10}
\caption{Within tournament association-game tradeoffs: later stages versus R16}
\end{figure}

As a final point in favor of our approach, we show that imposing the association constraint at earlier stages of the tournament increases the likelihood of same-nation match-ups in the subsequent rounds. Using our estimated model of goal outcomes and the at-most-one same-association match mechanism, we assess the predicted change in the number of same-nation matches in later stages of the tournament. We find that for every same-association pairing generated in the R16, there is a 0.10 reduction in the same-nation games in later stages of the tournament. In Figure 10 we illustrate these two compensating effects in all seasons between 2004 and 2018.

6. Conclusion

We document a constrained-assignment problem—with huge public interest and millions of euro in prize money at stake in the draw outcomes—where the randomization components need to be transparent. In the main body of the paper, we evaluate the chosen assignment rule both theoretically and empirically. Using the precise theoretical structure for the matching procedure and a commonly-used structural model for football match outcomes, we quantify the distortions caused by the competition’s association constraint. Even though this, however, cannot be determined, since “the identification of design constraints is usually more difficult because they are rarely communicated by the organizers” (Csató, 2018).

\textsuperscript{33}
the constraint on same-nation matches has significant repercussions on the matching probabilities, we show that the existing UEFA solution is close to a constrained-best. While alternative mechanisms can shift the probabilities of particular match-ups, there is no clear way to do so in a mechanism that treats all equal teams equally.

In the final section of the paper, we quantify the effects from weakening the association constraint. In particular, we allow for at most one out of eight R16 pairings to be same nation. This modification reduces the distortions by approximately 70 percent, where we find similar effect sizes both over R16 match probabilities and expected tournament prizes.
References


Roth, Alvin E. “The economist as engineer: Game theory, experimentation, and computation as tools for design economics,” Econometrica, 2002, 70 (4), 1341–1378.


A.1. **Proof of Proposition 2.** The constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw is distinct from: (i) The mechanism that fairly draws over $\Gamma$; (ii) The dynamic mechanism that fairly draws admissible pairs; and (iii) The constrained $\mathcal{W}$-to-$\mathcal{R}$ dynamic draw.

A.1.1. **Part (i).** Consider a problem of matching $\mathcal{R} = \{a, b, c, d\}$ to $\mathcal{W} = \{e, f, g, h\}$ under the set of constraints $H = \{ae, bf, cg, dh, ah, bg, dg\}$. The three resulting feasible matchings are given by $V_1 = \{ag, be, ch, df\}$, $V_2 = \{ag, bh, ce, df\}$, and $V_3 = \{ag, bh, cf, de\}$. Under the fair draw from $\Gamma = \{V_1, V_2, V_3\}$, the probability of $V_1$ is equal to $\frac{1}{3}$. However, under the dynamic $\mathcal{R}$-to-$\mathcal{W}$ mechanism, (and knowing that $a$ is degenerate) the probability of $V_1$ is given by:

$$Pr\{V_1\} = Pr\{be \in V^*\} = \sum_{x \in \mathcal{W}} Pr\{w_1 = x\} \cdot Pr\{be \in V^* | w_1 = x\} = \frac{13}{36}.$$ 

Hence, the two mechanisms are distinct.

A.1.2. **Part (ii).** Consider a problem of matching $\mathcal{R} = \{a, b, c, d\}$ to $\mathcal{W} = \{e, f, g, h\}$ under the set of constraints $H = \{bf, cg, ch, bg, dh\}$. Among the five resulting feasible matchings, only $V_1 = \{af, bh, ce, dg\}$ contains the match $af$. Under the dynamic $\mathcal{W}$-to-$\mathcal{R}$ mechanism, the probability of $V_1$ is $\frac{161}{864}$; However, under the dynamic $\mathcal{R}$-to-$\mathcal{W}$ mechanism $Pr\{V_1\} = Pr\{af \in V^*\} = \frac{55}{288}$. Hence, the two mechanisms are distinct.

A.1.3. **Part (iii).** Consider a problem of matching $\mathcal{W} = \{a, b, c, d\}$ to $\mathcal{R} = \{e, f, g, h\}$ under the set of constraints $H = \{bf, cg, ch, bg, dh\}$. Among the five resulting feasible matchings, only $V_1 = \{af, bh, ce, dg\}$ contains the match $af$. Under the dynamic draw of pairs $Pr\{V_1\} = Pr\{af \in V^*\} = \frac{59}{308}$; However, under the dynamic $\mathcal{R}$-to-$\mathcal{W}$ mechanism $Pr\{V_1\} = Pr\{af \in V^*\} = \frac{55}{288}$. Hence, the two mechanisms are distinct.

A.2. **Additional Results.**

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34 The proof requires at least a $4 \times 4$ market, as a $3 \times 3$ is degenerate with the standard symmetric group constraints and a single asymmetric same-nation exclusion. The proof can obviously be extended to any $n \times n$ market by making the remaining $(n-3)$ match partners unique through exclusions.

35 The proof becomes more cumbersome but still goes through if we removed the $dg$ exclusion that forces $ag$ to be degenerate.

36 A similar counterexample can be constructed with the standard group restriction enforced, but would require a $5 \times 5$ market; We omit it for tractibility and instead, focus on a $4 \times 4$ sub-market.

37 The four other matchings are: $V_2 = \{ae, bh, cf, dg\}$; $V_3 = \{ag, bh, ce, df\}$; $V_4 = \{ag, bh, cf, de\}$; $V_5 = \{ah, be, cf, dg\}$. 

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A.2.1. Simulation Errors.

**Proposition 4.** Simulating the mechanism $10^6$ times leads to 95 percent confidence intervals smaller than ±0.001.

**Proof.** Assignments are independent draws from a fixed distribution with a probability of selecting assignment $V$ given by $f(V)$. The probability that the particular match $ab$ is selected is given by $p_{ab} = \sum_{V \in M(ab)} f(V)$ where $M_{ab} := \{ V \in \Gamma | ab \in \mu \}$ is the set of matchings which include $ab$. We simulate the vector $8^2$-vector $\hat{p}$ where each element in $\hat{p}_{ab}$ is calculated from the $N$ independent simulation assignments $(\hat{V}_i)_{i=1}^N$

$$\hat{p}_{ab} := \frac{1}{N} \sum_{i=1}^N \mathbf{1} \left\{ ab \in \hat{V}_i \right\} .$$

The vector $\hat{p}$ has the obvious property that $\mathbb{E}(\hat{p}) = p$. We can use the central-limit theorem to show that $\sqrt{n}(\hat{p} - p) \xrightarrow{D} \mathcal{N}_{64}(0, \Omega)$ for the variance-covariance matrix $\Omega$ has a generic element given by:

$$\omega_{ab,cd} = \Pr \{ ab \land cd \} - \Pr \{ ab \} \Pr \{ cd \} ,$$

which can be estimated by

$$\hat{\omega}_{ab,cd} = \frac{1}{N} \sum_{i=1}^N \mathbf{1} \left\{ ab, cd \in \hat{V}_i \right\} - \hat{p}_{ab} \hat{p}_{cd} .$$

However, given a simulation-size of $N = 10^6$, a conservative estimates (as $\omega_{ab,ab} \leq \frac{1}{4}$) for the 95 percent confidence interval for each probability $p_{ab}$ is given by $\hat{p}_{ab} \pm \frac{1.96}{2000} \approx \hat{p}_{ab} \pm 0.001$. □
## Table 4. Format of the post-qualifying stages of the UCL between 1956 and 2018

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<th>K1</th>
<th>K2</th>
<th>G1</th>
<th>G2</th>
<th>K3</th>
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</table>

**Note:** K1, K2, and K3 denote the 1st, the 2nd, and the 3rd knock-out round, respectively. G1 and G2 denote the 1st and the 2nd group stage, respectively. QF denotes the quarterfinal, SF the semifinal, and F the final game.

## Table 5. Number of teams from each association participating in the UCL R16 by season

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**Mean:** 3.4 3.1 1.5 2.1 2.1

**Note:** BEL indicates Belgium, CYP Cyprus, CZE Czech Republic, DEN Denmark, ENG England, ESP Spain, FRA France, GER Germany, NED the Netherlands, ITA Italy, POR Portugal, RUS Russia, SCO Scotland, SUI Switzerland, TUR Turkey, UKR Ukraine. TOP5 denotes English, Spanish, French, German, and Italian associations together.
Table 6. Number of same-nation exclusions generated by each national association in the UCL R16 by season

<table>
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<th>Season</th>
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</tr>
<tr>
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<td>9</td>
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<td>17</td>
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<tr>
<td>2018</td>
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<td>2</td>
<td></td>
<td>1</td>
<td>7</td>
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</tr>
<tr>
<td>Mean</td>
<td>2.8</td>
<td>2.8</td>
<td>1.0</td>
<td>1.7</td>
<td>5.5</td>
<td>5.6</td>
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</tr>
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</table>

Note: (1) indicates a constraint generated by FC Zenit (RUS) and FC Dynamo Kyiv (UKR).

Table 7. Same-nation exclusions in the UCL R16 by season

<table>
<thead>
<tr>
<th>Season</th>
<th>ENG</th>
<th>ESP</th>
<th>FRA</th>
<th>GER</th>
<th>ITA</th>
<th>POR</th>
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</tr>
<tr>
<td>2006</td>
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</tr>
<tr>
<td>2007</td>
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<td>2</td>
<td>1</td>
<td>3</td>
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</tr>
<tr>
<td>2010</td>
<td></td>
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<tr>
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<td></td>
</tr>
<tr>
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<td>1</td>
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<tr>
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<td>2</td>
<td>2</td>
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<td>2014</td>
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<td>2</td>
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<tr>
<td>2015</td>
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<td>2</td>
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<td>2</td>
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<tr>
<td>2016</td>
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<td>2017</td>
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<td>1</td>
<td>2</td>
<td>2</td>
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<td>1</td>
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<td>2018</td>
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<td>1</td>
<td>1</td>
<td>2</td>
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<td>1</td>
</tr>
</tbody>
</table>

Note: In a (m|n)-pair m indicates the number of seeded (group stage winners) teams and n the number of unseeded (group stage runners-up) teams. m × n is the total number of exclusions generated by a given association.
Table 8. Estimated bivariate Poisson model coefficients by season

<table>
<thead>
<tr>
<th>Year</th>
<th>$\mu$</th>
<th>$\eta$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>-0.25</td>
<td>0.42</td>
<td>-2.06</td>
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<tr>
<td>2005</td>
<td>-0.48</td>
<td>0.46</td>
<td>-2.</td>
</tr>
<tr>
<td>2006</td>
<td>-0.71</td>
<td>0.57</td>
<td>-1.88</td>
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<tr>
<td>2007</td>
<td>-0.87</td>
<td>0.55</td>
<td>-1.99</td>
</tr>
<tr>
<td>2008</td>
<td>-0.48</td>
<td>0.46</td>
<td>-2.02</td>
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<tr>
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<td>-0.88</td>
<td>0.37</td>
<td>-1.71</td>
</tr>
<tr>
<td>2010</td>
<td>-1.34</td>
<td>0.29</td>
<td>-1.37</td>
</tr>
<tr>
<td>2011</td>
<td>-1.47</td>
<td>0.31</td>
<td>-1.33</td>
</tr>
<tr>
<td>2012</td>
<td>-0.64</td>
<td>0.27</td>
<td>-11.5</td>
</tr>
<tr>
<td>2013</td>
<td>-0.25</td>
<td>0.34</td>
<td>-2.23</td>
</tr>
<tr>
<td>2014</td>
<td>-0.05</td>
<td>0.31</td>
<td>-13.86</td>
</tr>
<tr>
<td>2015</td>
<td>-0.04</td>
<td>0.27</td>
<td>-15.91</td>
</tr>
<tr>
<td>2016</td>
<td>-0.09</td>
<td>0.34</td>
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<tr>
<td>2017</td>
<td>-0.07</td>
<td>0.39</td>
<td>-15.46</td>
</tr>
<tr>
<td>2018</td>
<td>-0.05</td>
<td>0.37</td>
<td>-16.12</td>
</tr>
</tbody>
</table>

Note: $\mu$ denotes the constant term, $\eta$ the home-effect parameter, and $\beta$ the correlation coefficient between the number of goals scored by the two opposing teams.
Figure 12. Values of the equal-treatment-of-equals objective function for the UEFA mechanism against the dynamic mechanism that fairly draws admissible pairs (2004–18)

Figure 13. Values of the equal-treatment-of-equals objective function for the UEFA mechanism against the dynamic \( W \)-to-\( R \) mechanism (2004–18)