Balanced Growth Approach to Forecasting Recessions

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Abstract
In this paper, we propose a hybrid alternative to Dynamic Stochastic General Equilibrium models with emphasis on forecasting performance at times of rapid changes (recessions). We interpret hypothetical balanced growth ratios as moving targets for economic agents that rely upon an Error-Correction Mechanism to adjust to changes in an underlying state Vector Autoregressive process. Our proposal is illustrated by an application to a pilot Real Business Cycle model for the US economy from 1948 to 2017. An extensive recursive validation exercise over the last 32 years, covering 3 recessions, is used to highlight the impressive parameters in variance and 1 to 4 step ahead forecasting performance of our hybrid model.

Key words: Hybrid model; VAR; DSGE; ECM; RBC
JEL classification: C53

1 Introduction
Dynamic Stochastic General Equilibrium (DSGE) models are generally justified on the grounds that they provide a structural foundation for policy analysis and are indeed widely used for that purpose. However, their forecasting failures in times of rapid changes (such as the 12/07-06/09 Great Recession) raise concerns relative to their relevance for policy recommendations in such times when they are most critically needed. Hence, a widely recognized need for greater diversification of the...
macroeconomics toolbox with models that focus on improved recession forecasting performance at the cost of loosening the theoretical straitjacket of DSGE models.

In the present paper, we propose a "hybrid" (Pagan, 2003) alternative to DSGE models that focuses on forecasting recessions or, more realistically, tracking them. Our proposal extensively relies upon a time series methodology pioneered by Sargent, Hendry and co-authors in the seventies (occasionally referred to as the "London School of Economics" approach). Subsequent developments in the theory and practice of cointegration and Error-Correction-Mechanisms (hereafter ECMs) provided an operational framework.

Recessions present a challenging environment for structural models since, as discussed further below, each post-war recession had different causes. These took the form of largely unanticipated shocks inducing distributional shifts that are difficult to accommodate within the expectational foundation of DSGE models (Hendry and Mizon, 2014a). In this respect Vector Autoregressive (hereafter VAR) reduced form models are more flexible and able to respond faster to large shocks. In particular, Hendry and Mizon (1993) implemented a modeling strategy starting from an unrestricted VAR and testing for cointegration relationships that would lead to a structural ECM. Most importantly, it follows that this ECM parsimoniously encompasses the initial VAR, a critical validation criteria developed by Hendry and co-authors in the eighties. See Hendry and Richard (1982, 1989), Mizon (1984), or Mizon and Richard (1986).

To a significant extent, we follow a similar approach with one important modification. As an alternative to a data driven identification of cointegration relationships, we derive them from a theory model, though one that is more flexible than DSGE models. Specifically, we assume that at any point of time, economic agents reason in terms of hypothetical balanced growth ratios recognizing that such targets vary over time, hence the need to error correct. Data consistency is then retrieved by modeling the target process as a dynamic (typically integrated) state VAR process. Essentially, our objective is that of producing a structurally interpretable ECM forecasting model.
that encompasses a benchmark VAR process. Ultimately, the model validation relies upon recursive estimation (for parameter invariance) and out-of-sample 1-4 step ahead recursive forecasts over an extended test period (covering the last 3 recessions for the pilot application described below).

Our paper is organized as follows: In Section 2, we provide a partial review of a very extensive literature on DSGE models and related modeling issues in order to set the scene for our own proposal. In Section 3, we provide a brief description of the idiosyncratic causes of the 11 most recent US recessions in order to highlight the challenging environment one faces when aiming to forecast economic downturns. In Section 4, we present a detailed generic description of the approach we propose. In Section 5, we provide an application to a pilot Real Business Cycle (RBC) model for the US postwar economy, detailing the successive modeling steps and documenting an extensive recursive validation exercise covering no less than the last 3 US recessions. In Section 6, we focus our attention on the latest Great Recession that was caused by a global financial crisis in combination with the collapse of the housing bubble. Our objective is that of analyzing whether the ex-ante incorporation of related series in the model could have improved early detection of the recession onset. We shall find that it is impossible to reliably estimate the potential impact of such series prior to the recession, a finding that strengthens the argument that recessions are triggered by large unexpected distributional shifts (Hendry and Mizon, 2014a). Section 7 concludes and discusses ideas for future research. A technical appendix presents a pseudo-code for the RBC application together with data description (full data, program, and an Online Appendix are available on https://sites.google.com/site/martaboczon).

2 Literature review

DSGE models have become the workhorses of modern macroeconomics, providing a rigorous structural foundation for policy analysis. However, as recognized by a number of authors, even before the onset of the Great Recession, their high degree of theoretical coherence ("continuous and perfect optimization", Sims (2007)) produces
dynamic structures that are typically too restrictive to capture the complexity of observed behavior, especially at times of rapid changes. In order to obtain tractable solutions, DSGE models assume a stable long-run equilibrium trend path for the economy (Muellbauer, 2016), which is precisely why they often fail to encompass more densely parametrized and typically non-stationary VAR processes. A useful discussion of the inherent trade-off between theoretical and empirical coherence can be found in Pagan (2003), with DSGE and VAR models at opposite corners.

Before the onset of the Great Recession, several authors had proposed innovative approaches linking VAR and DSGE models. Juselius and Franchi (2007) translated assumptions underlying a DSGE model into testable assumptions on the long run structure of a cointegrated VAR model. Building upon an earlier contribution of Ingram and Whiteman (1994), Sims (2007) discussed the idea of combining a VAR style behavioral model with a Bayesian prior distribution that pulls the VAR style toward the shape of a linearized DSGE model. Formal implementations of that concept can be found in Smets and Wouters (2005, 2007), or Del Negro and Schorfheide (2008). See also An and Schorfheide (2007) for a survey of Bayesian methods used to evaluate DSGE models and an extensive list of related references.

Smets and Wouters (2007) also incorporated several types of frictions and shocks into a small DSGE model of the US economy. They showed that their model is able to compete with Bayesian VAR model in out-of-sample predictions. Along similar lines, Chari et al. (2007, 2009) proposed a method, labeled Business Cycle Accounting (BCA), that introduces frictions ("wedges") in a benchmark prototype model. BCA is advocated as a way of identifying classes of mechanisms through which "primitive" shocks lead to economic fluctuations. The use of wedges has since been criticized for lacking structural justification and identification, and ignoring the fundamental shocks (e.g. financial) that drive the wedge process. See e.g. Christiano and Davis (2006) or, in his lively language, Romer (2016). Nevertheless, BCA highlights a critical empirical issue which is that structurally invariant stationary DSGE models are not flexible enough to accommodate rapid changes induced by unexpected shocks.
to the system. Lack of structural invariance is a critical issue that we revisit in our approach.

Obviously, the debate about the limitations of DSGE models took a new urgency following their widespread predictive failures on the occasion of the 2008-2009 recession. Major policy models, such as the Bank of England Quarterly Model (Harrison et al. (2005)) were either abandoned or extensively revised. The subsequent discussion focused on the inability of DSGE models to respond to unexpected shocks. A few references are Caballero (2010), Castle et al. (2010, 2016), Hendry and Mizon (2014a,b), Hendry and Muellbauer (2018), and Stiglitz (2018). On the other hand, Christiano et al. (2018) offered a detailed and more upbeat discussion of how DSGE models have evolved in response to the Great Recession. See also Schorfheide (2011) for an insightful discussion of advances and remaining challenges in the DSGE literature.

More directly relevant to the object of our paper, there has been a growing emphasis on the need for DSGE models to share the scene with alternative approaches. We have already mentioned the concept of "hybrid" models in Pagan (2003). See also Wieland and Wolters (2012), Blanchard (2016), and Korinek (2017). The following quote from Trichet (2010), Governor of the Bank of France from 1993 to 2003 and President of the European Central Bank from 2003 to 2011, deserves our full attention: "But relying on judgment inevitably involves risks. We need macroeconomic and financial models to discipline and structure our judgmental analysis. How should such models evolve? The key lesson I would draw from our experience is the danger of relying on a single tool, methodology, or paradigm. Policymakers need to have input from various theoretical perspectives and from a range of empirical approaches. Open debate and a diversity of views must be cultivated - admittedly not always an easy task in an institution such as a central bank. We do not need to throw out our DSGE and asset-pricing models: rather we need to develop complementary tools to improve the robustness of our overall framework." We cannot think of a better motivation for the approach we propose in our paper.
In summary, we have tried to offer a brief overview of alternative views as to the future of DSGE models. Not being professional policy analysts, we do not intend to take side in that lively debate. Instead, as time series econometricians, we offer a "hybrid" alternative to DSGE models that has superior forecasting performance in times of rapid changes such as recessions, at the cost of a less rigid theoretical foundation. Along the words of Trichet (2010), we aim at offering an empirically performant complement, by no means a substitute, to DSGE models.

3 US Postwar Recessions

As discussed e.g. in Hendry and Mizon (2014a), a key issue with macroeconomic forecasting models is that of whether or not recessions constitute "unanticipated location shifts". More specifically, while one can generally identify indicators leading to a recession, the relevant econometric issue is that of whether or not such indicators can be incorporated ex-ante into the model and, foremost, whether or not their potential impact can be reliably estimated prior to each recession onset. As our initial attempt to address this fundamental issue, we provide a brief survey of the most likely causes for each of the US postwar recessions.

The 1945 recession was caused by the demobilization and the resulting transition from a wartime to a peacetime economy at the end of the Second World War. The separation of the Federal Reserve from the US Treasury is presumed to have caused the 1951 recession. The 1957 recession was likely triggered by an initial tightening of the monetary policy between 1955 and 1957, followed by its easing in 1957. Similar circumstances led to the 1960 recession. The 1969 recession was likely caused by initial attempts to close the budget deficits of the Vietnam War followed by another tightening of the monetary policy. The 1973 recession is commonly believed to originate from an unprecedented rise of 425% in oil prices, though many economists believe that the blame should be placed instead on the wage and price control policies of 1971 that effectively prevented the economy from adjusting to market forces. The main reason of the double dip recession of the 1980s is believed to be an ill-timed
FED monetary policy aimed at reducing inflation. Large increases in federal funds rates achieved that objective but also led to a significant slow down of the economic activity. There are several competing explanations for the 1990 recession. One was another rise of the federal funds rates to control inflation. The oil price shock following the Iraqi invasion of Kuwait and the uncertainties surrounding the crisis were likely contributing factors. Solvency problems in the savings and loan sector have also been blamed. The 2001 recession is believed to have been triggered by two unprecedented factors, September 11 and the collapse of the dot-com bubble. Last but not least, the Great Recession was caused by a global financial crisis in combination with the collapse of the housing bubble.

In summary, each postwar recession was triggered by idiosyncratic sets of circumstances, including but not limited to ill-timed monetary policies, oil shocks to aggregate demand and supply, and financial and housing crises. As we shall discuss further in Section 6 in the context of the Great Recession, such a variety of unique triggers makes it largely impossible to reliably econometrically estimate their potential impact prior to the actual onset of each recession, a conclusion that supports the words of Trichet (2010), as quoted in Section 2.

Table 1: Economic indicators: growth rate of real GDP ($\Delta \ln Y$), real gross private domestic investment ($\Delta \ln I$), and real personal consumption expenditures ($\Delta \ln C$) measured between the peak and through of the 11 US postwar recessions.

<table>
<thead>
<tr>
<th>Peak</th>
<th>Through</th>
<th>$\Delta \ln Y$</th>
<th>$\Delta \ln I$</th>
<th>$\Delta \ln C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1948Q4</td>
<td>1949Q4</td>
<td>-1.5%</td>
<td>-24.1%</td>
<td>3.4%</td>
</tr>
<tr>
<td>2 1953Q2</td>
<td>1954Q2</td>
<td>-2.4%</td>
<td>-10.6%</td>
<td>0.7%</td>
</tr>
<tr>
<td>3 1957Q3</td>
<td>1958Q2</td>
<td>-3.0%</td>
<td>-15.7%</td>
<td>-0.5%</td>
</tr>
<tr>
<td>4 1960Q2</td>
<td>1961Q1</td>
<td>-0.1%</td>
<td>-9.0%</td>
<td>-0.3%</td>
</tr>
<tr>
<td>5 1969Q4</td>
<td>1970Q4</td>
<td>-0.2%</td>
<td>-6.4%</td>
<td>1.7%</td>
</tr>
<tr>
<td>6 1973Q4</td>
<td>1975Q1</td>
<td>-3.1%</td>
<td>-24.8%</td>
<td>-0.7%</td>
</tr>
<tr>
<td>7 1980Q1</td>
<td>1980Q3</td>
<td>-2.2%</td>
<td>-15.9%</td>
<td>-1.2%</td>
</tr>
<tr>
<td>8 1981Q3</td>
<td>1982Q4</td>
<td>-2.6%</td>
<td>-25.3%</td>
<td>2.7%</td>
</tr>
<tr>
<td>9 1990Q3</td>
<td>1991Q1</td>
<td>-1.4%</td>
<td>-9.3%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>10 2001Q1</td>
<td>2001Q4</td>
<td>0.4%</td>
<td>-6.9%</td>
<td>2.1%</td>
</tr>
<tr>
<td>11 2007Q4</td>
<td>2009Q2</td>
<td>-4.0%</td>
<td>-29.4%</td>
<td>-2.4%</td>
</tr>
</tbody>
</table>
In spite of different causes, key economic indicators have responded to the previous recessions in a similar manner (see Table 1 above). Specifically, investment has always declined by more than output and in all but one postwar recessions, output has declined by more than consumption. Actually, the growth rate of consumption has remained positive in five of the 11 recessions, and the growth rate of output has never dropped below -4% with an average of -1.8% and a standard deviation of 1.3%.

4 Hybrid forecasting models

4.1 Outline

Since our main interest lies in forecasting macroeconomic aggregate variables with emphasis on times of rapid changes, it is essential to rely upon series that are neither detrended nor filtered in order to avoid "mistaken influences about the strength and dynamic patterns of relationships" (Wallis (1974)). Thus, we shall rely upon real seasonally adjusted per capita series, in order to avoid a need for seasonal dummies and to account for population growth. It is immediately apparent that these series are non-stationary (which is why they are frequently filtered to accommodate the DSGE stationarity constraint). Thus, cointegration provides a powerful tool for robustifying inference (Juselius and Franchi (2007)) and, from the agents' perspective, to avoid systematic mistakes and adjust to changes (Hendry (2018)).

As stated in the introduction, our primary objective is that of producing VAR-ECM forecasting models whose performance is to be recursively tested over an extensive validation period (1985:3 - 2017:3 for the RBC model presented below). Our approach belongs to a class of "Type II Hybrid" models as described in Pagan (2003) in that it relaxes the theoretical structure of DSGE models in order to improve their empirical, specifically forecasting, coherence. Our proposed approach consists of two stages.

In stage 1 (core model), we derive hypothetical balanced growth solutions. Not only are these much easier to compute than DSGE decision rules but, foremost, we would argue that they provide more obvious reference points for economic agents in a world
where statistics such as growth rates, saving ratios, and unemployment rates are widely available. Under our proposal, these balanced growth solutions serve two main purposes: on the modeling side, we shall interpret them as the agents’ cointegrated ECM moving targets; empirically, since these solutions clearly vary over time, they require the introduction of dynamic state variables. However, instead of introducing frictions (wedges) in the model equations, we shall let some key structural parameters vary over time and treat them as state variables. In particular, with reference to the RBC model presented below, empirical evidence from our use of unfiltered data unambiguously rejects the assumption that the capital share of output and the consumers’ preference for consumption relative to leisure have remained constant from 1948 to 2017. Therefore, the objective of stage 1 is that of producing fitted trajectories for the selected state variables that best match the observed trajectories of the hypothetical core balanced growth solutions.

In stage 2, we construct the actual forecasting model in the form of a dynamic state-space model consisting of two parts: (1) a (non-stationary) VAR process for the state variables; and (2) a (measurement) process in the form of an ECM for the log changes of the relevant macroeconomic variables as a function of changes in the state variables and, foremost, lagged differences between cointegrated log ratios and their target balanced growth solutions.

It is important to note that all modeling decision in stages 1 and 2 are to be ultimately evaluated in terms of recursive parameter invariance and forecasting performance over an extended validation period (1985-2017 for our RBC application). The reason for emphasizing both is that, while forecast performance remains our primary objective, forecasting in the presence of (suspected) structural breaks raises significant complications such as the selection of estimation windows. See e.g. Pesaran et al. (2006) or Pesaran and Timmermann (2007). In addition, we want to verify that the estimated trajectories of the state variables make sense from an economic viewpoint.

The pilot RBC model presented below illustrates the feasibility and, foremost, the
outstanding parameter invariance and forecasting performance of our proposed approach.

4.2 Implementation

In this section, we describe how we propose to implement stages 1 and 2.

Stage 1: The specification and estimation of the core model proceeds in several steps:

1. The core model specifies the components of a balanced growth optimization problem, essentially objective function(s) and accounting identities. It does not assume that the economy reaches such a balanced growth equilibrium at any point of time, only that agents aim for one based on their current perception of a vector of state variables (including but not limited to a tentative growth rate) that vary over time, hence the notion of chasing moving targets.

2. Period $t$ solutions to the agents’ optimization problems produce two complementary sets of first order conditions. The first set consists of balanced growth ratios between the decision variables, to be subsequently reinterpreted as moving cointegrated targets. The second set provides laws of motion for the individual variables that would converge towards a balanced growth equilibrium under a hypothetical scenario, whereby the state variables would remain constant over time. These two sets of conditions are denoted as:

$$
\begin{pmatrix}
    r^*_t \\
    \Delta x^*_t
\end{pmatrix}
= \begin{pmatrix}
    h_1 (s_t; \lambda) \\
    h_2 (s_t, s_{t-1}; \lambda)
\end{pmatrix},
$$

(1)

where $r^*_t \in \mathbb{R}^p$ denotes log ratios, $\Delta x^*_t \in \mathbb{R}^{p+1}$ log changes, $\lambda$ a vector of time invariant structural parameters, and $s_t \in \mathbb{R}^q$ a state vector yet to be determined. The superscript "*$\) is used to denote balanced growth solutions. Absence of that superscript denotes observed counterparts. Potential exogenous variables are omitted for the ease of notation.

3. As already mentioned and as illustrated further below, once we use data that
are neither detrended nor filtered, it is immediately obvious that \( r_t \) and, to a lesser extent, \( \Delta x_t \) vary considerably over time, especially on the occasion of recessions and recoveries, and that these variations cannot be adequately accounted for by letting only the growth rate vary over time. As discussed further below in the context of our RBC pilot application, there exists independent empirical evidence that some key structural parameters are not time invariant. Therefore, we allow for an appropriate subset of structural parameters to vary over time and to be included in \( s_t \) instead of \( \lambda \). We prefer our approach to the BCA alternative of introducing frictions (wedges) in equations of the model since, in particular, it preserves the possibility of considering policy interventions that would directly target these (easily interpretable) time varying parameters. Moreover, adjustment costs (frictions) are implicitly accounted for within our ECM framework. As already stated, both the selection of \( s_t \) and the final calibration of \( \lambda \) is ultimately based on recursive parameter invariance and forecasting performance of the VAR-ECM model. Therefore, all computations described next are conditional upon an arbitrarily value of \( \lambda \) and are recursive in the following sense. Let \( T \) denote the actual sample size and define a validation period \([T_a, T]\). Then, for any given \( T_\ast : T_a \to T \), we only use data up to \( T_\ast \) for estimation. Next, we compute forecasts for observations \( t \in [T_\ast + 1, T_\ast + 4] \), which for \( t \leq T \) are compared with the actual data for model validation and calibration of \( \lambda \). For the ease of notation, we delete most explicit reference to \( T_\ast \) in the derivations that follows.

4. The next step consists of estimating the state trajectories that are subsequently used for the ECM component of the model. In order to construct trajectories that match the balanced growth core solutions, we rely upon a sequential Non-Linear Least-Squares (NLLS) procedure. Specifically, for any given value of \( \lambda \) we compute sequential point estimates for \( \{s_t\}_{t=1}^T \) as follows

\[
\hat{s}_t (\lambda) = \arg\min_{s_t} ||\epsilon_t (s_t, \hat{s}_{t-1} (\lambda)); ||_2, \quad t : 1 \to T
\]  

(2)
under an appropriate Euclidean L-2 norm and where \( \epsilon_t (\cdot) \) stands as a shorthand notation for the differences in equation (1). Note that \( \{ \hat{s}_t (\lambda) \}_{t=1}^T \) only depends on \( \lambda \). Thus, it only needs to be computed once for \( t : 1 \to T \), even though only values up to \( T^* \) are used for recursive estimation.

Stage 2: The VAR-ECM forecasting model:

1. Following estimation of \( s_t \), we specify a state VAR\((l)\) process for \( \hat{s}_t \), say

\[
\hat{s}_t = A_0 + \sum_{i=1}^{l} A_i \hat{s}_{t-i} + \epsilon_t, \tag{3}
\]

where \( \epsilon_t \sim \mathcal{IN} (0, \Sigma_A) \). For the RBC application described below, we ended selecting \( l = 2 \). Let \( \Gamma = (A_0, \{A_i\}, \Sigma_A) \) denote the VAR parameters.

2. Next, we specify an ECM model of the form

\[
\Delta x_t^o = D_0 + D_1 \Delta \hat{s}_t + D_2 \Delta x_{t-1}^o - D_3 (r_{t-1} - h_1 (\hat{s}_{t-1}; \lambda)) + v_t, \tag{4}
\]

where \( \Delta x_t^o = \Delta x_t - h_2 (\hat{s}_t, \hat{s}_{t-1}; \lambda) \) and \( v_t \sim \mathcal{IN} (0, \Sigma_D) \).

Additional regressors can be included as needed for improving forecast accuracy. This particular parametrization of the ECM guarantees that, as long as \( D_0 \) is zero, it would converge to a balanced growth equilibrium under a hypothetical scenario whereby the state variables (or estimates thereof) would remain constant over time. In practice, omitted variables and/or measurement errors in the state variables could produce non-zero estimates for \( D_0 \). In the results presented below for the RBC model, we find the estimated \( D_0 \) coefficients to be very small (though significantly different from zero), which is definitely good news for the theoretical coherence of our approach. We shall also find that the \( D_3 \) adjustment coefficients are significant with the correct ECM signs.
5 Pilot application to a RBC model

5.1 Model specification

In order to test both the feasibility and prediction performance of our approach, we reconsider a baseline RBC model taken from Rubio-Ramirez and Fernandez-Villaverde (2005) and subsequently re-estimated by DeJong et al. (2013) as a conventional DSGE model, using HP filtered per capita data. It consists of a representative household that maximizes a discounted utility flow from consumption $c_t$ and leisure $l_t$. The core balanced growth solution solves the problem

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\varphi} l_t^{1-\phi}}{1-\phi} \right)^{1-\phi}$$

subject to the constraints

$$y_t = k_t^\alpha (n_t z_t)^{1-\alpha}, \quad n_t = 1 - l_t, \quad \Delta \ln z_t = g, \quad k_{t+1} = (y_t - c_t) + \delta k_t$$

where $y_t, c_t, k_t$ denote real per capita (unfiltered) seasonally adjusted quarterly output, consumption, and capital, $n_t$ per capita weekly hours as a percentage of discretionary time, and $z_t$ stochastic productivity; $\alpha$ denotes the capital share of output, $\beta$ the household discount rate, $\varphi$ the relative importance of consumption versus leisure, $\phi$ the degree of relative risk aversion, and $1 - \delta$ the depreciation rate of capital. Utility maximization under the hypothetical balanced growth scenario $\Delta \ln z_t = g$ produces the following expressions for the two balanced growth ratios

$$r^* = \left( \frac{\ln \left( \frac{y}{c} \right)}{\ln \left( \frac{y}{c} \cdot \frac{1-n}{n} \right)} \right) = \left( \frac{\ln \left( \frac{p(s;\lambda)}{q(s;\lambda)} \right)}{d - \ln (1 - \alpha)} \right) = h_1 (s; \lambda)$$

where $s = (g, d, \alpha)$ and $\lambda = (\beta, \delta, \phi)$ according to the partition that will be justified below when we select the state variables, and where

$$p(s;\lambda) = \frac{1}{\alpha} \left[ \frac{1}{\beta} \exp \left( g \left( 1 + \frac{\phi + e^d}{1 + e^d} \right) \right) - \delta \right], \quad q(s;\lambda) = p(s;\lambda) - [e^g - \delta],$$
\[ and \quad d = \ln\left(\frac{1 - \varphi}{\varphi}\right). \quad (9) \]

Next, let
\[ \Delta x = \left( \Delta \ln\left(\frac{y}{n}\right), \Delta \ln\left(\frac{c}{n}\right), \Delta \ln\left(\frac{1 - n}{n}\right) \right)', \quad (10) \]

and notice that under balanced growth, we would have \( \Delta x^* = (g, g, 0)' \). However, inspection of the data immediately reveals that none of the five components in \( r \) and \( \Delta x \) are anywhere close to being constant. The time variations of the log ratios \( r_t \) are illustrated in Figure 1. Lesser variations are observed for \( \Delta x_t \). It is obvious that \( g \)

![Figure 1: Balanced growth ratios. The dotted line (with vertical axis on the left) denotes \( \ln\left(\frac{y_t}{c_t}\right) \), and the solid line (with vertical axis on the right) denotes \( \ln\left[\frac{(1 - n_t) / n_t \times y_t / c_t}{y_t / c_t}\right]. \)]

cannot be constant but it also appears that, at a minimum, \( d \) and \( \alpha \) cannot be both constant. Supporting evidence that \( \alpha \) is not constant over time is provided by the share of capital series retrieved from Federal Reserve Bank of St. Louis (FRED) that we illustrate in Figure 4 (see Section 5.6), together with our own estimate. We don’t have direct corroborating evidence for \( d \). However, our estimated state trajectory for \( \varphi_t \) (relative preference for consumption versus leisure) is broadly comparable with analyses of hours worked in the US. Juster and Stafford (1991) document a reduction in hours per week between 1965 and 1981, whereas Wilson and Jones (2018) report a
modest increase between 1979 and 2016, resulting from increased women labor participation. We considered two alternative choices for \( s_t \): \((g_t, \alpha_t)\) and \((g_t, d_t, \alpha_t)\). We found that the latter choice clearly dominates the first one both in terms of fit with the actual \((r_t, \Delta x_t)\) series in equations (7) to (10) and, foremost, predictive performance over the validation period (1985:3 - 2017:3). This is not surprising as we would find hard to believe that the household relative preference for consumption versus leisure and the share of capital have remained constant from 1948 to 2017, a time period that has witnessed extraordinary technological and preference changes and covers 11 recessions. Therefore, all expressions in equations (7) to (9) are now time indexed except for the time-invariant parameter vector \( \lambda \). In particular, equation (7) now represents the period \( t \) agents’ moving balanced growth targets. The corresponding laws of motion are given by

\[
\begin{pmatrix}
\Delta \ln \left( \frac{y_t}{n_t} \right) \\
\Delta \ln \left( \frac{c_t}{n_t} \right) \\
\Delta \ln \left( \frac{1-n_t}{n_t} \right)
\end{pmatrix} =
\begin{pmatrix}
g_t + \Delta \left( \frac{\alpha_t}{\alpha_{t-1}} \ln p_t \right) \\
g_t + \Delta \left( \frac{1}{\alpha_t} \ln p_t + \ln q_t \right) \\
\Delta \left( d_t - \ln (1-\alpha_t) - \ln \left( \frac{p_t}{q_t} \right) \right)
\end{pmatrix} = h_2 (s_t, \hat{s}_{t-1}; \lambda),
\]

where \( p_t \) and \( q_t \) are short-hand notations for \( p(s_t; \lambda) \) and \( q(s_t; \lambda) \) in equation (8).

These theoretical laws of motion offer useful guideline for the specification of the stage 2 ECM model in equation (4). Additionally, for theoretical coherence, we propose to specify the ECM in such a way that it would converge to a balanced growth equilibrium under a hypothetical scenario whereby \( s_t \) would remain constant. After extensive experimentation and forecasting validation, we settled on the following ECM specification

\[
\Delta x_t^o = D_0 + D_1 \Delta \hat{s}_t + D_2 \Delta x_{t-1}^o - D_3 (r_{t-1} - h_1 (\hat{s}_{t-1}; \lambda)) + v_t,
\]

where \( \Delta x_t^o = \Delta x_t - h_2 (s_t, \hat{s}_{t-1}; \lambda) \) and the difference \( r_{t-1} - h_1 (\hat{s}_{t-1}; \lambda) \) represents the ECM correction term. Note that as long as \( D_0 \) remains small, this ECM meets our converge criteria.
5.2 Data

As previously discussed, we rely on data that are neither detrended nor filtered. In order to construct the data on real consumption per capita ($c_t$), we divide the sum of real consumption expenditures in services and non-durable goods by the working-age population. Second, we obtain the data on real output per capita ($y_t$) by dividing the sum of real consumption expenditures (in services and non-durable goods) and real gross private domestic investments by the working-age population. Note that both $c_t$ and $y_t$ are further divided by 4 to account for annualization. Finally, we compute the fraction of time dedicated to work ($n_t$) by dividing total hours worked in the US economy by three factors: (1) our measure our population, (2) $16 \times 7 = 112$, assuming a daily average of 16 hours of a discretionary time, and (3) 52 to account for the fact that the data on hours worked are seasonally-adjusted on an annualized-level basis.

All the aforementioned data series (except for hours worked) are retrieved from the St. Louis Fed's Federal Reserve database (FRED). Quarterly data on consumption and investments are seasonally adjusted and in billions of chained 2009 dollars. Quarterly data on population is not seasonally adjusted and in thousands of persons. The data on hours worked comes from the Office of Productivity and Technology of the U.S. Bureau of Labor Statistics. Quarterly hours worked are seasonally-adjusted on an annualized-level basis and presented in billions. The FRED codes for the data are listed in Table 4 in the Appendix, together with graphs for ($y_t, c_t, n_t$) in Figure 9.

5.3 Recursive estimation and forecasting procedure

In this section, we provide a brief description of our recursive estimation and forecasting procedure. Actual results are reported in the relevant subsections that follow.

All estimation results are computed recursively over the validation period 1985:3 to 2017:3, which covers 32 years and 3 recessions, including the latest Great Recession. It represents an impressive 44.6% of our full sample. We rely upon the NLLS procedure described above in Section 4.2 in order to construct the state VAR process. Recursive
parameter estimates for the ECM are computed using a customized SURE subroutine. All estimation results are be provided in the form of figures for the recursive estimates together with 95% confidence intervals. Recursive forecasts are obtained by Monte Carlo simulations, whereby for each $T_a \in [T_a, T - 4]$ 1 to 4 steps ahead VAR and ECM errors are drawn at random 1,000 times and used to produce nominal 95% intervals. Computing the percentage of times actual values fall within these intervals, we shall find a close match between nominal and actual percentages. In addition to figures highlighting recursive forecast accuracy, we also provide Root Mean Square Percentage Error (RMSPE) summary statistics defined as

$$f_{i,j} = 100 \cdot \frac{1}{T - T_a} \left[ \sum_{t=T_a+1}^{T} \left( \frac{\hat{x}_{i,t-1}^j - x_t^j}{x_t^j} \right)^2 \right],$$

(13)

where $\hat{x}_{i,t-1}^j$ denotes the period $t - i \ (i : 1 \to 4)$ MC mean forecast for $x_t^j \ (j : 1 \to 3)$.

We have also used the relative root-mean-square error (RMSE) with overall similar results. Depending upon an eventual decision context, more specific evaluation criteria could be implemented. See e.g. Elliott and Timmermann (2016).

Last but not least, since we strongly advocate full replicability, the complete data set, our program, and additional results are available on our website.

5.4 VAR benchmark

As we mentioned in the introduction, the empirical performance of DSGE models is frequently dominated by that of VAR processes. Therefore, we first present full recursive results obtained from a VAR(2) process for $\Delta x_t$:

$$\Delta x_t = Q_0 + Q_1 \Delta x_{t-1} + Q_2 x_{t-2} + w_t.$$  

(14)

Recursive 1-4 step ahead VAR forecasts are presented in Figures 10-12 in the Appendix.
Overall, these results are impressive and, as expected, present a challenging benchmark for our hybrid VAR-ECM model. They also highlight an important issue that also applies to the VAR-ECM model. Specifically, it is widely recognized that the 2007-2009 Great Recession was triggered by a major financial crisis and, foremost, one that had no precedent. However, neither the VAR benchmark nor our VAR-ECM model include a financial sector. Nevertheless, the VAR impressive forecasting performance suggests that the impact of the Great Recession on the real sector (output, consumption, and labor) was by no means atypical, though definitely of larger magnitude than for earlier recessions. In Section 6, we investigate whether the addition of financial series into either the VAR or ECM components of our model can improve forecasting the Great Recession. We shall find that it cannot for the simple reason that there is not enough covariation before the recession to estimate the potential impact of these series.

5.5 Calibration of \((\beta, \delta, \phi)\)

As discussed in Section 5.1, we selected \((g_t, d_t, \alpha_t)\) as our state variables, which leaves \((\beta, \delta, \phi)\) as our remaining structural parameters. The latter are calibrated, rather than formally estimated, by looking at a combination of criteria: foremost, the VAR-ECM forecasting performance (relative to that of the VAR benchmark), but also parameter recursive invariance as well as the signs and order of magnitude of the error correction coefficients \(D_3\) in Equation (12). Accounting for the fact that recursive performance according to equation (13) was largely unchanged across neighboring values, we selected \((\beta, \delta, \phi) = (0.97, 0.98, 1.3)\) on the basis of the two other criteria. These values are in line with those widely used in the literature. All the results reported below are derived under these values.

5.6 The state VAR model

First, we need to compute values for the \(\{\hat{s}_t(\lambda)\}_{t=1}^{T}\) according to equation (2). After experimentation, we found that, in terms of the overall model performance, it is best to use for \(\hat{g}_t\) the actual growth rate of the principal component of \(c_t\) and \(y_t\), and to
apply our NLLS criterion to compute \( \{\hat{d}_t(\lambda), \hat{\alpha}_t(\lambda)\}_{t=1}^T \) given \( \{\hat{g}_t\}_{t=1}^T \).

Next, we (recursively) estimate an unconstrained VAR(2) process for \( \{\hat{g}_t, \hat{d}_t, \hat{\alpha}_t\} \). LM test statistics for residual autocorrelation and/or higher order coefficients are insignificant. As expected, the VAR(2) has two (near) unit roots. The best fit obtains for \( \hat{d}_t \) with recursive \( R^2 \)'s equal to 98\% throughout the entire validation period. Unsurprisingly, in view of Figure 2, the worst fit obtains for \( \hat{g}_t \), with \( R^2 \)'s in the range from 0.10 to 0.16. As \( \{\hat{\alpha}_t\}_{t=1}^T \) trends upward, we note that its recursive \( R^2 \)'s increase with \( T \) from 0.17 to 0.68.

Figure 2: Estimated trajectory of \( g_t \). The dotted line denotes NLLS estimates of \( g_t \), and the solid line denotes fitted values of \( g_t \) resulting from an unrestricted SURE estimation of the state VAR model.

In Figures 2-4, we illustrate the initial estimates and fitted values (for \( T = 278 \)) for \( (g_t, \varphi_t, \alpha_t)_{t=1}^T \), where \( \varphi_t = (\exp(d_t) + 1)^{-1} \). In addition, we include for comparison the share of capital series constructed as \( 1 - \omega_t \), where \( \omega_t \) denotes the labor share in nonfarm business sector retrieved from FRED. We note similar variation though our estimates are lower by about 10\%, but we are not discussing exactly the same macroeconomic series.
Figure 3: Estimated trajectory of $\varphi_t = (\exp(d_t) + 1)^{-1}$. The dotted line denotes NLLS estimates of $\varphi_t$, and the solid line denotes fitted values of $\varphi_t$ resulting from an unrestricted SURE estimation of the state VAR model.

Figure 4: Estimated trajectory of $\alpha_t$. The dotted line denotes NLLS estimates of $\alpha_t$, the solid line denotes fitted values of $\alpha_t$ resulting from an unrestricted SURE estimation of the state VAR model, and the dashed line denotes the FRED data.

5.7 The ECM model

The ECM component of our hybrid model is of the form presented in Equation (12). We use our restricted SURE program to compute recursive estimates. This resulted
in the elimination of two regressors. One is the ECM factor associated with \( \ln \left( \frac{1-n}{n} \right) \), which leaves \( \ln \left( \frac{y}{c} \right) \) as the sole ECM target. The other is \( \Delta \hat{d}_t \). Both eliminations are meaningful. Equilibrium adjustments in \( n_t \) are clearly affected by factors beyond the control of agents. As for the elimination of \( \Delta \hat{d}_t \), it is likely due to the fact that the quarterly variations of \( \hat{d}_t \) are generally quite small so that they have a minimal impact on \( \Delta y_t \). The recursive VAR and ECM parameters estimates are illustrated in the Online Appendix and exhibit impressive invariance over a 32 years validation period covering no less than 3 recessions.

The signs and order of magnitude of the ECM adjustment coefficients are meaningful (see Figure 8). Specifically, the quarterly ECM adjustments toward equilibrium are of the order of 8%, suggesting a relatively rapid adjustment to the target movements, likely a key factor in the model quick response to recessions - and also one that would guarantee quick convergence to a balanced growth equilibrium were \( s_t \) to remain constant for a few years. The relatively low values of \( \beta \) combined with the ECM correction \( (D_3) \) appear to support the Deaton (1991) and Carroll (2000) argument that consumers have shorter horizon than frequently thought.

The VAR-ECM recursive 1-4 step ahead forecasts are illustrated in Figures 5-7. They are visually very close to those of the VAR benchmark (see Figures 10-12 in the Appendix).

The most impressive characteristic of the 1-step ahead VAR-ECM forecasts is that, as was the case for the VAR benchmark, they closely track the onset of the Great Recession and the subsequent recovery even in the absence of a financial sector. The same can be said of the 4-step ahead forecasts though with a 1 to 3 quarters lag. Overall, the recursive forecasting performance of our hybrid VAR-ECM model is impressive. In sharp contrast with most DSGE models, it stands against a VAR benchmark and, foremost, closely (ex-ante) tracks the Great Recession. In terms of the hybrid trade-off discussed in Pagan (2003), it appears that we have succeeded in preserving a significant degree of theoretical coherence at a fairly small loss of
empirical (forecasting) coherence.

Figure 5: Out-of-sample recursive forecasts for real output per capita (in 10,000 of chained 2009 dollars) generated by the VAR-ECM model. We illustrate the 1 to 4 step ahead forecasts from the top to the bottom panel. The solid lines denote the mean forecast calculated over 1,000 MC repetitions, the shaded areas denote the corresponding 95% confidence intervals, and the dotted lines indicated the data.
Figure 6: Out-of-sample recursive forecasts for real consumption per capita (in 10,000 of chained 2009 dollars) generated by the VAR-ECM model. We illustrate the 1 to 4 step ahead forecasts from the top to the bottom panel. The solid lines denote the mean forecast calculated over 1,000 MC repetitions, the shaded areas denote the corresponding 95% confidence intervals, and the dotted lines indicated the data.
Figure 7: Out-of-sample recursive forecasts for the fraction of time spent working generated by the VAR-ECM model. We illustrate the 1 to 4 step ahead forecasts from the top to the bottom panel. The solid lines denote the mean forecast calculated over 1,000 MC repetitions, the shaded areas denote the corresponding 95% confidence intervals, and the dotted lines indicated the data.
Figure 8: VAR-ECM model: Recursive parameter estimates of the ECM factor associated with \( \ln(y_c) \) in the ECM regression of \( \Delta x_t^{(1)} \) (top panel), \( \Delta x_t^{(2)} \) (middle panel), and \( \Delta x_t^{(3)} \) (bottom panel) according to the specification given in equation (12), where \( \Delta x_{t-1} = [\Delta x_{t-1}^{(1)}, \Delta x_{t-1}^{(2)}, \Delta x_{t-1}^{(3)}]' \). The solid lines represent the recursive parameter estimates and shaded areas the corresponding 95% confidence intervals.
Table 2: Relative mean absolute error summary statistics for 1-4 step ahead recursive out-of-sample forecasts for $y_t$ (top panel), $c_t$ (middle panel), and $n_t$ (bottom panel) computed for the VAR-ECM model (column 2) and the benchmark VAR (column 3).

<table>
<thead>
<tr>
<th>Forecast horizon</th>
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<th>VAR</th>
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<tr>
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<td>$y_t$</td>
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<tr>
<td>1-step</td>
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<td></td>
<td>$c_t$</td>
<td></td>
</tr>
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<td>0.37%</td>
</tr>
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<td>2-step</td>
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<td>0.58%</td>
</tr>
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</tr>
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</tr>
<tr>
<td></td>
<td>$n_t$</td>
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</tr>
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</tr>
<tr>
<td>4-step</td>
<td>1.86%</td>
<td>1.61%</td>
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6 The Great Recession

In order to further improve our out-of-sample 1-4 step ahead forecasts, we tried to incorporate into our model additional information from a sizable set of publicly available macroeconomic variables. Specifically, from the FRED database, we retrieved a total of 100 quarterly time series starting between 1948Q1 and 1973Q1 related to assets and liabilities of commercial banks, money stock measures, interest rates, new residential sales and construction, capacity utilization, employment situation, as well as house price, business activity, and consumer confidence indices (see Online Appendix for a detailed list of variables). Among the aforementioned series, we focused primarily on leading financial and housing variables in the hope of improving our 3 and 4 step ahead forecasts of the onset of the Great Recession.

We have incorporated the additional FRED series into our hybrid model by introducing their fourth lags into either the VAR equation or (as the first difference) the ECM equation. We have experimented with single variables as well as subsets. Moreover, in
the VAR equation, we allowed for different FRED series in each of the state equations for added flexibility and to avoid overparameterization.

Our primary objective was to find out whether by adding any of these series to our model, we could improve early detection of the Great Recession onset in the form of a reduction of the lags in 3 and 4 step ahead forecasts. Accessorily, we wanted to see whether such additions could also reduce the RMSE of our forecasts.

Table 3: Relative mean absolute error summary statistics for 3 and 4 step ahead out-of-sample forecasts for \( y_t \), \( c_t \), and \( n_t \) computed for the benchmark VAR-ECM model (w/o) and the VAR-ECM model with a fourth lag of a single FRED series incorporated into the VAR equation (w/).

<table>
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<th>Series</th>
<th>Start date</th>
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<th>( c_t )</th>
<th>( n_t )</th>
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<tr>
<td>1</td>
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<td>1.98%</td>
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<tr>
<td></td>
<td></td>
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</tr>
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<tr>
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</tr>
<tr>
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<td>1.94%</td>
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</tr>
<tr>
<td>3</td>
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<td>2.04%</td>
<td>2.05%</td>
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</tr>
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<tr>
<td>4</td>
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<tr>
<td>7</td>
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<tr>
<td>9</td>
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Our findings relative to our primary objective are uniformly negative in that, even after extensive experimentation, we are essentially unable to reduce the 3 and 4 step
ahead forecasting gaps. As for the secondary objective of reducing the forecast errors, we present in Table 3 summary statistics for the ten most representative series. Only the inclusion of the consumer index into the VAR equation generates reductions in MEA, ranging from 6% to 9%, which might not be that surprising for a model of consumer behavior. On the other hand, the other nine (financial and housing) series present a fairly consistent picture: they all moderately decrease MEAs for $n_t$ but do worse for $c_t$ and $y_t$. The latter finding clearly supports the concept of an "unanticipated location shift" Hendry and Mizon (2014a). At the current stage of our research, it definitely appears as if it remains impossible to ex-ante quantify the impact of the unfolding financial and housing crises on the (RBC) recession onset.

7 Conclusion

While our paper presents research that is still in progress, the results we report are very promising. Our balanced growth approach has produced an RBC application that achieves an operational compromise between theoretical and empirical coherence at a minimum loss of forecasting performance relative to that of a VAR benchmark model. Our model exhibits impressive recursive parameter invariance and 1 to 4 step ahead forecasting performance over an extensive validation period covering 32 years (44% of our sample) and 3 most recent recessions.

On the theory side, the model fully validates the notion that agents reason in terms of balanced growth ratios and adjust rapidly to changes thereof. Foremost, the estimated model has the key property that it would converge to a balanced growth equilibrium if ever the state variables were remain constant.

On the empirical side, the forecasting performance of the model results from our finding that, beyond their long term trend, the state variables $d_t$ and $\alpha_t$ exhibit similar variation patterns across recessions that were triggered by very different (unexpected) circumstances. Moreover, though varying, $d_t$ and $\alpha_t$ have clear structural interpretations and are amenable to policy interventions.
Obviously, our initial success needs to be confirmed in broader applications. While there currently appears to be limited scope to incorporate ex-ante into the model variables linked to specific recession triggers, there ought to be room to take advantage of the cyclical behavior of the state variables, e.g. in the form of our next project on tension indices similar to the one proposed by DeJong et al. (2005). Alternatively, following the call for input "from a range of empirical approaches" (Trichet, 2010), we could rely upon independent assessments or signals of increasing likelihood of a recession to implement ex-ante counterfactual analyses of the potential impact of such a recession on the economic sector being modeled. Therefore, we strongly believe that the initial success of our approach paves the way for future exciting developments.

Acknowledgments

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References


Appendix: Pseudo code for the RBC application

Structural parameters: $\lambda' = (\beta, \delta, \phi)$. State variable: $s_t' = (g_t, d_t, \alpha_t)$. Observables: $(y_t, c_t, n_t)$.

1. Compute analytical balanced growth solutions:

   $r_t^* = \begin{bmatrix} \ln \left( \frac{y_t^*}{c_t^*} \right), \ln \left( \frac{y_t^*}{c_t^*} \frac{1-n_t^*}{n_t^*} \right) \end{bmatrix}' = h_1(s_t; \lambda)$

   $\Delta x_t^* = \begin{bmatrix} \Delta \ln \left( \frac{y_t^*}{n_t^*} \right), \Delta \ln \left( \frac{c_t^*}{n_t^*} \right), \Delta \ln \left( \frac{1-n_t^*}{n_t^*} \right) \end{bmatrix}' = h_2(s_t, s_{t-1}; \lambda)$

Denote the actual data counterparts without the * superscript.

2. Start calibration loop:

   2.1. Select $\lambda$

   2.2. Estimate state variables:

   $\hat{s}_t(\lambda) = \text{argmin}_{s_t} ||\epsilon_t(s_t, \hat{s}_{t-1}(\lambda); \lambda)||_2, \quad t : 1 \to 278,$

   where $\epsilon_t'(s_t, s_{t-1}; \lambda) = [r_t - h_1(s_t; \lambda), \Delta x_t - h_2(s_t, s_{t-1}; \lambda)].$ Alternatively, set $g_t = \Delta (\text{principal component of } y_t \text{ and } c_t)$ and optimize in $(d_t, \alpha_t | g_t)$.

3. Start recursive loop (given $\lambda$):

   3.1. Set $T = 150$

   3.2. Estimate state VAR for $\{\hat{s}_t\}_{t=1}^T$:

   $\hat{s}_t = A_{0,T} \hat{s}_{t-1} + A_{1,T} \hat{s}_{t-1} + A_{2,T} \hat{s}_{t-2} + u_t, \quad u_t \sim \mathcal{N}(0, \Omega_T).$

   Store $\{\hat{A}_{i,T}\}_{i=0}^2$ and $\hat{\Omega}_T$. Note: SURE and/or unit root alternatives make little difference.
3.3. Estimate SURE ECM for \( \{\Delta x_t\}^T_{t=1} \):

\[
\Delta x_t^o = D_{0,T} + D_{1,T} \Delta \hat{s}_t + D_{2,T} \Delta x_{t-1}^o - D_{3,T} (r_{t-1} - h_1 (\hat{s}_{t-1}; \lambda)) + v_t,
\]

where \( \Delta x_t^o = \Delta x_t - h_2 (\hat{s}_t, \hat{s}_{t-1}; \lambda) \) and \( v_t \sim \mathcal{N} (0, \Sigma_T) \).

Store \( \{\hat{D}_{j,T}\}^3_{j=0} \) and \( \hat{\Sigma}_T \).

3.4. Conduct Monte Carlo forecast simulation \( (k : 1 \to 1,000) \):

i. Forecast 4-step ahead from VAR \( \{\hat{s}_{T+l|T}\}^4_{l=1} \).

ii. Forecast 4-step ahead from ECM: \( \{\Delta \hat{x}_{T+l|T}\}^4_{l=1} \) given \( \{\hat{s}_{T+l|T}\}^4_{l=1} \).

iii. Recover and store \( \{\hat{x}_{T+l|T}\}^4_{l=1} \).

4. If \( T < 278 \), then \( T = T + 1 \) and go to 3.2. Else, end recursive loop.

5. Evaluate recursive performance. For \( T : 150 \to 278 \):

5.1. Graph \( \{\hat{A}_{i,T}\}^3_{j=0} \) and \( \{\hat{D}_{j,T}\}^3_{j=0} \).

5.2. Graph MC 95% confidence intervals for \( l : 1 \to 4 \) step-ahead forecasts of \( x_{T+l} \).

6. As needed, return to 2.1 and select a different value of \( \lambda \).
### Appendix: Tables and Figures

Table 4: Names and codes of FRED series used in the estimation of VAR-ECM model.

<table>
<thead>
<tr>
<th>FRED series name</th>
<th>FRED code(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Real personal consumption expenditures: Services</td>
<td>DSERRA3Q086SBEA, PCESVC96</td>
</tr>
<tr>
<td>2 Real personal consumption expenditures: Nondurable goods</td>
<td>DNDGRA3Q086SBEA, GPDIC1</td>
</tr>
<tr>
<td>3 Real Gross Private Domestic Investment</td>
<td>PCNDGC96</td>
</tr>
<tr>
<td>4 Civilian Noninstitutional Population: 25 to 54 years</td>
<td>LNU00000060</td>
</tr>
</tbody>
</table>
Figure 9: Data used in the estimation. Top panel: real output per capita (in 10,000 of chained 2009 dollars), middle panel: real consumption per capita (in 10,000 of chained 2009 dollars), bottom panel: fraction of time spent working.
Figure 10: Out-of-sample recursive forecasts for real output per capita (in 10,000 of chained 2009 dollars) generated by the VAR benchmark model. We illustrate the 1 to 4 step ahead forecasts from the top to the bottom panel. The solid lines denote the mean forecast calculated over 1,000 MC repetitions, the shaded areas denote the corresponding 95% confidence intervals, and the dotted lines indicated the data.
Figure 11: Out-of-sample recursive forecasts for real consumption per capita (in 10,000 of chained 2009 dollars) generated by the VAR benchmark model. We illustrate the 1 to 4 step ahead forecasts from the top to the bottom panel. The solid lines denote the mean forecast calculated over 1,000 MC repetitions, the shaded areas denote the corresponding 95% confidence intervals, and the red lines indicated the data.
Figure 12: Out-of-sample recursive forecasts for the fraction of time spent working generated by the VAR benchmark model. We illustrate the 1 to 4 step ahead forecasts from the top to the bottom panel. The solid lines denote the mean forecast calculated over 1,000 MC repetitions, the shaded areas denote the corresponding 95% confidence intervals, and the dotted lines indicated the data.